Math 5700 Homework #11

(1) Prove DeMoivre's Theorem using induction.

$$z^n = (r(\cos\theta + i\sin\theta))^n = r^n(\cos(n\theta) + i\sin(n\theta)),\ \forall n \in \mathbb{N} \text{ and } \forall z \in \mathbb{C}$$

(2) Change these complex numbers to trigonometric form.

(a) $z = -4i$

(b) $z = -5 + 4i$

(c) $z = 2 + 3i$

(3) Plot and change these complex numbers to rectangular form.

(a) $5(\cos 30^\circ + i\sin 30^\circ)$

(b) $7\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

(4) Prove this claim.

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then $z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$.

(5) Calculate these complex numbers using DeMoivre's Theorem and give the answer in rectangular form.

(a) $(\sqrt{3} + i)^7$

(b) $\left(\frac{1-i}{\sqrt{2}}\right)^{20}$

(c) $(2\cos 210^\circ + 2i\sin 210^\circ)^5$

(6) We proved DeMoivre's Theorem for $n \in \mathbb{N}$. Make a conjecture about $\sqrt[n]{\cos\theta + i\sin\theta}\ \forall n \in \mathbb{N}$ and give three examples that support your claim.

(7) Prove that the opposite of complex number $z = r(\cos\theta + i\sin\theta)$ is given by $-z = r(\cos(\theta + \pi) + i\sin(\theta + \pi))$.

(8) Let $z = r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$.

(a) Sketch *z, iz,* and *z/i*.

(b) What is the geometric effect of multiplying a complex number by *i*? What is the geometric effect of dividing a complex number by *i*?

(9) Using the trigonometric form of a complex number, find $z\bar{z}$.