Determine if each series is absolutely convergent, conditionally convergent, or divergent. Justify your answers.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{n\left(7^{n+1}\right)}$

2. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{5n^2+1}{n^3+n^4}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{200^n\, n!}{(3n)!}$

4. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{n}+\frac{2}{n^4}\right)$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2 3^{n+1}}{e^{2n}}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{4\sqrt{n}}{n^3+5}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}(-4)^n}{(2n+1)!}$

8. $\displaystyle\sum_{i=1}^{\infty} \frac{3i}{\sec\left(i^2\right)}$

9. $\displaystyle\sum_{n=11}^{\infty} \left(\left(\frac{n-10}{n}\right)^n - 1\right)$

10. $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+5^n}{8^n}$

Rewrite each sum or number as an infinite series, if necessary. Then prove the series converges, and find the exact sum.

11. $\ln\left(\frac{2}{5}\right)+\ln\left(\frac{5}{8}\right)+\ln\left(\frac{8}{11}\right)+\ln\left(\frac{11}{14}\right)+...$

12. $0.51212121212....$

13. $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+5^n}{8^n}$

Find the convergence set for these power series. (Remember to check the endpoints.)

14. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n!}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n n^4 (x-8)^n}{(n+2)!}$

16. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{n+1}(3n+5)}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^n (x-3)^n}{5n+1}$

Find the Taylor Series for these functions, centered at the given a-value.

18. $f(x)=\dfrac{16}{2-x}-1$, a = 1

19. $f(x)=\dfrac{-2x^5}{2-8x}$, a = 0

20. $f(x)=\sin x$, a = $\dfrac{\pi}{3}$

21. $f(x)=e^{2x}$, a = ln(2)

22. $f(x)=e^{2x}$, a = 0

23. $f(x)=x^3+2x^2-4x+1$, a = 5

24. $f(x)=5\ln(4-x)+1$, a = 3

25. Consider the following four infinite series.

(a) $\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty}2^n$

(c) $\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{n^{1/2}}$

(d) $\displaystyle\sum_{n=1}^{\infty}2^{-n}$

Which of the following statements is true?

(A) (a), (b), (c) and (d) all converge.
(B) (a) and (b) converge, but (c) and (d) diverge.
(C) (a) and (c) converge, but (b) and (d) diverge.
(D) (a) and (d) converge, but (b) and (c) diverge.
(E) (a) converges, but (b), (c) and (d) diverge.
(F) (a) diverges, but (b), (c) and (d) diverge.

26. I rode my first century (a.k.a. 100 miles) bike ride last summer in 6 hours of riding time (360 minutes), and this summer, I rode the same ride in 5 hours 40 minutes (340 minutes). Last week, after an encounter with a vampire, I have now realized that I will live in this body forever. After making peace with this fact, I decided my current goal is to decrease my annual century bike ride time by the same percentage/fraction as my progress this year indicated. If I achieve this goal, how many hours of my infinitely long life will be spent riding these annual century bike rides, including the first ride?

27. Wanting an extra puzzle to solve, seven calculus students divide a cake as follows: First, they divide it into eight equal pieces, each taking an eighth of the cake. Then, they divide the leftover eighth into eighths, each taking an eighth and so on. How much of that yummy cake does each person get?