Math 5700 Homework #9

Solve these equations (give exact answers).

(1) $2(3^{x+1})=1+\frac{5}{3^x}$

(2) $\ln(x-3)+\ln(x-2)=\ln(2x+24)$

(3) $(\log x)^2-11\log x=-10$

(4) $\ln(e^x)-2\ln e=\ln(e^4)$

(5) $\log(x+1)-\log(x-1)=1$

(6) $2(\log x)^2+\log x^5=3$

(7) $e^{-\ln x}=2$

(8) $\frac{4}{2+e^x}=\frac{6}{3+e^{-x}}$

(9) $\frac{3^x-3^{-x}}{3^x+3^{-x}}=9$

(10) $4^{2x-1}=3^{2x+3}$

(11) $4^{x+1}+4^{1-x}=10$

(12) $18^{2x}3^{-2x}=6$

Simplify these expressions.

(13) $\log_5(\log_5(\log_5 625))$

(14) $\log_{\frac{1}{4}}\left(\frac{16^2}{2^{-3}}\right)$

(15) $\frac{\ln(a^b)-\ln(a^c)}{b-c}$

(16) $(\ln x)(\log_x 5)$

(17) $5^{3\log_5 1-2\log_5 4}$

(18) $\ln\left(2e^{x^2+3x}\right)$

Solve these inequalities.

(19) $|1-e^{2x}|\le 5$

(20) $\log_4 x+\log_4(x-1)<\log_4 6$

(21) $\log_2 x+\log_2(x-6)\ge 4$

(22) $e^{x^2-1}\le 1$

(23) $\log(x-3)>\log(x+6)-1$

(24) Find the inverse function for $f(x)=\frac{e^x-e^{-x}}{2}$.

(25) If *a, b, c, d* are positive real numbers such that $a + b + c + d = 2$, then $X = (a + b)(c + d)$ is bounded by which two consecutive natural numbers?

(26) Which of these numbers is a perfect square in every possible base, b, for that number?
36_b , 144_b , 81_b , 225_b

(27) If f is a function such that $f(x-1)=x^2-3x+5$ then what is $f(x+1)$?

(28) Given a collection of three numbers, the smallest is zero. If the mean of the three numbers is log(4) and the median is log(5), what is the largest number?

(29) If $\log_8(\log_4(\log_2 x))=0$, then what is $x^{-\frac{2}{3}}$?

(30) Jose speaks the truth and nothing but the truth every other day. On the other days, he always lies. Today he made exactly four of the following statements. Which statement did he not make today?

(a) Half of my friends are male.
(b) Three of my friends are older than I.
(c) I always speak the truth.
(d) 288 is divisible by 12.
(e) I have a prime number of friends.