Math5700
Proof Homework

Name: ____ Date: ____________________

Please staple this sheet (as a cover sheet) to your homework.

1. For the Fibonacci sequence, defined recursively as
   $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$, I claim the direct formula is
   $$a_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{\sqrt{5}\ 2^n} \text{ for all } n = 1, 2, 3, \ldots$$

Prove this.

2. Prove that for all natural numbers n, $n^2 - n$ is even.
3. Prove $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
4. Prove that there are infinitely many primes.
5. Make a conjecture about the sum $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!}$ and prove your claim.

6. For f given recursively by $f(0) = 0$, $f(n) = f(n-1) + 3n + 2$ for all n = 1, 2, ... find an explicit formula for $f(n)$ and prove your formula is valid.

7.

Suppose that we draw on a plane n lines in "general position" (i.e. with no three concurrent, and no two parallel). Let s_n be the number of regions into which these lines divide the plane, for example, $s_3 = 7$ in Figure 8.1.

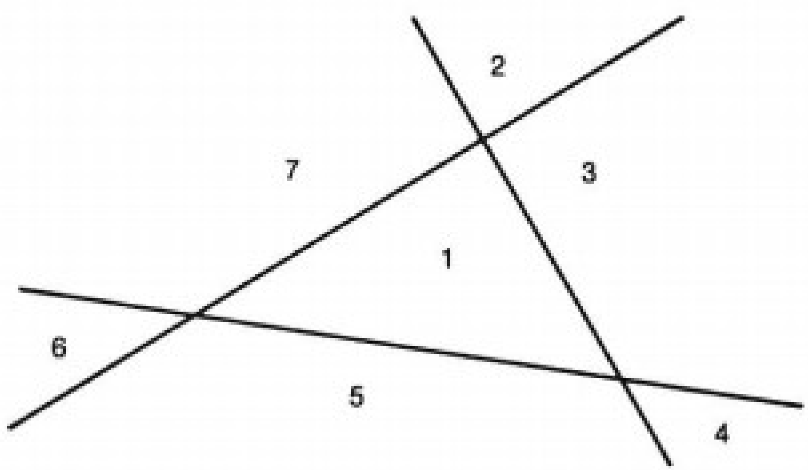

Figure 8.1

(a) By drawing diagrams, find s_1, s_2, s_3, s_4 and s_5
(b) From these results, make a conjecture about a formula for s_n
(c) Prove this formula by mathematical induction.