

Math1210 (Calculus 1) Review (Graph sketching & story problems)

- For $f(x) = \frac{2x^3}{(x-1)^3}$ (with $f'(x) = \frac{-6x^2}{(x-1)^4}$ and $f''(x) = \frac{12x+12x^2}{(x-1)^5}$), answer the following questions.
 - Find all asymptotes.
 - Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.
- A metal cylindrical container with an open top is to hold 64π cubic feet of space. If there is no waste in construction, find the radius and height of the cylinder which requires the least amount of material.
- Suppose a spherical bubble is decreasing in size. Its radius is decreasing at a constant rate, changing from 6 cm to 4 cm in 5 minutes. How fast is the volume changing when the radius is 3 cm?
- A 10 foot ladder is leaning against a vertical wall. If the ladder is sliding down the wall at a constant rate of 0.5 ft/sec, how fast is the bottom end of the ladder moving away from the wall when top of the ladder is 6 feet above the ground?
- For $f(x) = (x^2 + 2)^2$, answer the following questions.
 - Find all asymptotes.
 - Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.
- For $f(x) = \frac{2x+1}{x+4}$, answer the following questions.
 - Find all asymptotes.
 - Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.
- Find two nonnegative numbers whose sum is 10 and whose product is a maximum.
- Find two numbers whose product is -16 and the sum of whose squares is a minimum.
- For $f(x) = (x-1)^3$, answer the following questions.
 - Find all asymptotes.
 - Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.

10. What are the dimensions of the cone with slant height 15 meters that has the largest volume? (*Note:* The formula for volume of a cone is given by $V = \frac{1}{3} \pi r^2 h$. And, slant height is the distance from the apex, i.e. the “point,” of the cone to any point on its circular base.)
11. A farmer wishes to fence off three identical adjoining rectangular pens, each with 200 square feet of area. Suppose that the outer boundary of the pens requires heavy fence that costs \$5 per foot, but that the two internal partitions require fence costing only \$3 per foot. What dimensions of each pen will produce the least expensive cost for the project?
12. Let $y = \cos(\pi x) + x^3$. If x changes from 1 to 1.05, approximately how much does y change?
13. For $f(x) = \frac{x-1}{x-4}$, answer the following questions.
- Find all asymptotes.
 - Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.
14. Sand is pouring from a pipe at the rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose height is always $\frac{1}{4}$ the diameter of the base, how fast is the height increasing when the pile is 4 feet high?
15. For $f(x) = 2x^3 - 9x^2 + 12x$, answer the following questions.
- Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.
16. For $f(x) = \frac{3(x+2)^2}{(x-1)^2}$ (given $f'(x) = \frac{-18(x+2)}{(x-1)^3}$ and $f''(x) = \frac{18(2x+7)}{(x-1)^4}$), answer the following questions.
- Find all asymptotes.
 - Fill in the sign line for $f'(x)$.
 - Find all local min and max **point(s)**, if they exist.
 - Fill in the sign line for $f''(x)$.
 - Find all inflection **point(s)**, if any exist.
 - Sketch the **entire** graph of the function using **all** this information.
17. Use differentials to approximate the increase in surface area of a soap bubble when its radius increases from 3 inches to 3.025 inches.
18. For all rectangles with a diagonal of 2 inches, find the dimensions of the one with the maximum area.
19. Find the dimensions of the right circular cylinder of greatest volume that can be inscribed in a right circular cone, if the cone has a radius of 6 inches and height of 12 inches. (*Note:* The volume of a cylinder is $V = \pi r^2 h$; the volume of a cone is $V = \frac{1}{3} \pi r^2 h$.)
20. The area of an equilateral triangle is decreasing at a rate of 2 square centimeters per second. Find the rate at which the length of a side is changing when the area of the triangle is $100\sqrt{3}$ square centimeters.