Math1210 Midterm 3 Extra Review Key

1. Evaluate

   (a) $\int \left(2x^4(x^5-1)^{-2/3}\right)dx = \frac{6}{5}(x^5-1)^{1/3}+C$

   (b) $\int \left(3\sqrt[5]{t}-\frac{4}{t^2}+2t^3-\sin t+10\right)dt = \frac{5}{2}t^{6/5}+\frac{4}{t}+\frac{1}{2}t^4+\cos t+10t+C$

   (c) $\int \frac{(2x+3)^2}{\sqrt{x}}dx = \frac{8}{5}x^{5/2}+8x^{3/2}+18\sqrt{x}+C$

   (d) $\int (4x^5-\cos x+\sqrt[3]{x^2})dx = \frac{2}{3}x^6-\sin x+\frac{3}{5}x^{5/3}+C$

   (e) $\int \frac{4x}{\sqrt{x^2-3}}dx = 4\sqrt{x^2-3}+C$

   (f) $\int (2x^3\sqrt{2x^4+3})dx = \frac{1}{6}(2x^4+3)^{3/2}+C$

2. Solve the following differential equation.

$$\frac{dy}{dx} = \frac{4x^3+\frac{1}{x^2}}{3y^4} \quad \text{such that} \quad y=-1 \quad \text{when} \quad x=1$$

Answer: $y=\sqrt[5]{\frac{5}{3}x^4-\frac{5}{3x}-1}$

3. For $f(x)=x^2+\frac{2}{x}$

   (a) Find all asymptotes, if they exist. VA: $x=0$, no HA, SA: $y=x^2$

   (b) Fill in the sign line for $f'(x)$

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   <------0-----1------>

   (c) Find all local minimum and maximum points, if they exist, or state that they DNE.
   Min at (1, 3); no max

   (d) Find the global minimum and maximum points, if they exist, or state that they DNE.
   No global min or max points

   (e) Fill in the sign line for $f''(x)$

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   <--------- $\sqrt[3]{-2}$ --------0---------->

   (f) Find all inflection points, if they exist, or state that they DNE.
   Inflection point at ($\sqrt[3]{-2}$, 0)

   (g) Sketch the graph of $f(x)$.

4. For the function $f(x)=\frac{3x-2}{x-5}$ on the closed interval [1, 4], decide whether or not the Mean Value Theorem for Derivatives applies. If it does, find all possible values of c. If not, then state the reason.

   Answer: Yes MVT applies because the function is continuous and differentiable on [1, 4].
   c = 3

5. Solve $x^4 - 53 = 0$ using Newton's Method, accurate to four decimal places.

Use $x_{n+1} = x_n - \frac{x_n^4 - 53}{4x_n^3} = \frac{4x_n^4 - x_n^4 + 53}{4x_n^3} = \frac{3x_n^4 + 53}{4x_n^3}$. If you start with $x_1 = 2.5$ (why? Because I know that $2^4 = 16 \text{ and } 3^4 = 81$ and 53 is somewhere between 16 and 81), then you'll get these values out: 2.5, 2.723, 2.698505497, 2.69816794, and 2.698167876. So the answer is approximately 2.6982 to four decimal places.

6. For $f(x) = 3x^2 + 4x - 1$ on [0, 2], decide whether or not the Mean Value Theorem (for Derivatives) applies. If it does, find all possible values of c. If not, then state the reason.

Answer: Yes MVT applies because the function is continuous and differentiable everywhere.
c = 1

7. Solve this equation using (A) the Bisection Method **and** (B) Newton's Method to three decimal places.

$f(x) = 2x^3 - 4x + 1 = 0$ On [0, 1]

Answer: should get (A) the midpoint of the interval from 0.2578125 to 0.26171875 which would be 0.259765625 which is about 0.2598 and (B) 0.2586

8. Solve this differential equation.

$\frac{dy}{dx} = \frac{x + 3x^2}{y^2}$ and $y = 2$ when $x = 0$

Answer: $y = \sqrt[3]{\frac{3}{2}x^2 + 3x^3 + 8}$

9. Evaluate $\sum_{i=1}^{10} [(i-2)(2i+5)] = 725$

10. Evaluate the definite integral **using the definition** (the tedious way).

$\int_{-1}^{2} (5x - 1)\,dx$. (Note: Here is the definition. $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$)

Answer: $\Delta x = \frac{3}{n}$, $x_i = -1 + \frac{3i}{n}$, $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left(\frac{-18}{n} + \frac{45i}{n^2} \right)$, $\int_{-1}^{2} (5x - 1)\,dx = 4.5$

11. Find $G'(x)$ given $G(x) = \int_4^x x^3 (t^2 - 2)\,dt$

Answer: $G'(x) = 3x^2 \int_4^x (t^2 - 2)\,dt + x^3 (x^2 - 2)$

12. Evaluate $\sum_{i=1}^{10} [(3i-4)(i+5)] = 1640$

13. Evaluate the definite integral **using the definition** (the tedious way). $\int_0^3 (4x^2 - 1)\,dx$.

Answer: $\Delta x = \frac{3}{n}$, $x_i = \frac{3i}{n}$, $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left(\frac{108i^2}{n^3} - \frac{3}{n} \right)$, $\int_0^3 (4x^2 - 1)\,dx = 33$

14. Find $G'(x)$ given $G(x) = \int_3^{\tan x} (t^3 - \sin(t^2))\,dt$

Answer: $G'(x) = (x^3 - \sin(x^2))\, sec^2 x$