1.1 Problem Solving

"It is important to approach problem solving with the attitude of meeting an intellectual challenge rather than finishing a menial task."

Polya's steps for Problem Solving

1. identify the problem
2. make a plan; devise a strategy for soln
3. implement the plan
4. reflect; decide if it solved the problem.

Problem-solving Strategies:

1. Guess and Test
2. draw a picture/diagram/table
3. use manipulatives
4. math principles (like order of operations)
5. look for a pattern
6. solve a simpler problem
7. work backwards
8. process of elimination
9. use algebra

Questions that encourage investigation and create deeper understanding:

1. Why? (or why not?)
2. What if...?
3. What patterns do I notice?
4. What predictions can I make?
5. How is this like (or different)...?
6. Will that always work?
7. Can I do it another way?
8. What other related problems might I explore?

Ex 1:
$$\begin{array}{r} SUN \\ +\ FUN \\ \hline SWIM \end{array}$$

$$\begin{array}{r} 1\_\_ \\ +\ \_\_\_ \\ \hline 10\_\_ \end{array}$$

S = 1
U =
N =
F =
W = 0
I =
M =

F = 8 or 9
W = 0 ~~or 1~~

try 1 F = 8

$$\begin{array}{r} 1 \\ 173 \\ +\ 873 \\ \hline 1046 \end{array}$$

4 or 5

U = 6, 7, ~~8~~
try U = 7
I = 4
N = 3

other idea:
$$\begin{array}{r} 134 \\ +\ 934 \\ \hline 1068 \end{array}$$

Ex 2: Find the sum 1 + 2 + 3 + ... + 100.

$1+100 = 101$
$2+99 = 101$
$3+98 = 101$
$\vdots$
$50+51 = 101$

$= 101(50) = 5050$

ex $1+2+3+\dots+191$

$= 192(95\frac{1}{2})$

$1+191 = 192$
$2+190 = 192$
$\vdots$
$95+97 = 192$
96

$1+2+3+\dots+n$

$= (1+n)\frac{n}{2}$

$\sim (1+n)n(\frac{1}{2})$

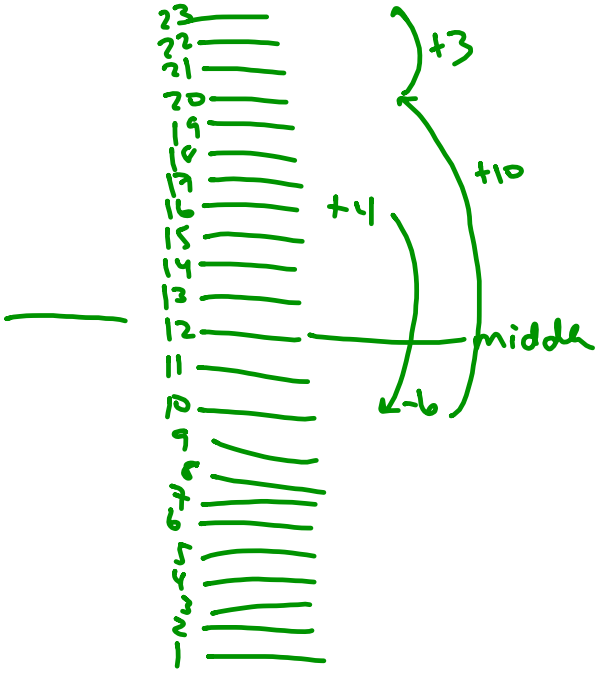
Ex 3: Five friends decided to give a party and split the costs equally. Al spent $4.75 on invitations, Betty spent $12.00 for drinks and $5.25 on vegetables, Carl spent $24.00 on pizza, Dani spent $6.00 on paper plates and napkins, and Ellen spent $13.00 for decorations. Determine who owes money to whom and how the money can be paid.

A	4.75
D	6.00
E	13.00
B	17.25
C	24.00
total:	65.00

amt each person should pay in total: $13

A owes	8.25	① A pays $4.25 to B
D owes	7.00	② A pays $4 to C
B gets	4.25	③ D pays $7.00 to C
C gets	11.00	

Ex 4: An elevator stopped at the middle floor of a building. It then moved up 4 floors, stopped, moved down 6 floors, stopped, and then moved up 10 floors and stopped. The elevator was now 3 floors from the top floor. How many floors does the building have?



Ex 5: We need exactly four gallons of water, but we only have a 5-gallon container and a 3-gallon container, with no measuring marks. How can you use those two containers to measure exactly four gallons?

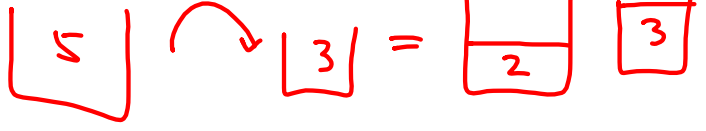
fill 5 gal → pour into 3 gal

→ dump 2 gal into separate container

repeat to get 4 gal in sep. container

w/ no separate container:

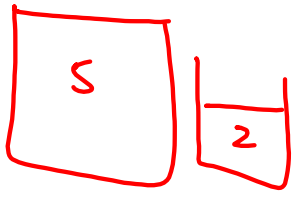
fill 5 gal



next: dump 3-gal ⇒

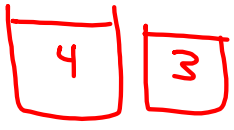
pour 2 gals in 3-gal bucket:

fill 5-gal bucket:



pour water from 5-gal bucket

⇒



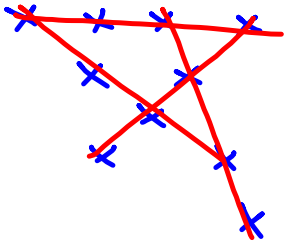
Ex 6: Find three consecutive Natural numbers whose sum is 78.

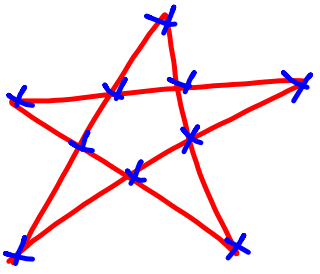
① $\frac{78}{3} = 26$ $\quad 25 + 26 + 27 = 78$

② w/ algebra $\quad x =$ first #

$$x + x+1 + x+2 = 78$$
$$3x + 3 = 78$$
$$3x = 75$$
$$x = 25$$

Ex 7: Arrange 10 people such that there are five rows each containing four people.





Ex 8: Show why 3 always divides evenly into the sum of any three consecutive whole numbers.

$4+5+6$

$x+x+1+x+2$

$=3x+3$

$=3(x+1)$

3 is a factor

Ex 9: Find the sum 58 + 59 + ... + 203.

58+203
59+202
60+201

$\not{\#}\frac{(203-58)}{2}(203+58) = 18922.5$

Note

$1+2+3+\ldots+n = \frac{n}{2}(n+1)$

$n+(n+1)+\ldots+m = \frac{(m-(n-1))}{2}(n+m)$

$1+2+\ldots+203 = \frac{203}{2}(203+1)$

$1+2+\ldots+57 = \frac{57}{2}(57+1)$

$58+59+\ldots+203 = (1+2+\ldots+203)-(1+2+\ldots+57)$

$= \frac{203}{2}(203+1) - \frac{57}{2}(57+1) = 203(102) - 39(57)$

$= 19,053$

Ex 10

$$47+48+\cdots+1196 = \frac{(1196-46)}{2}(47+1196)$$
$$= 714{,}725$$

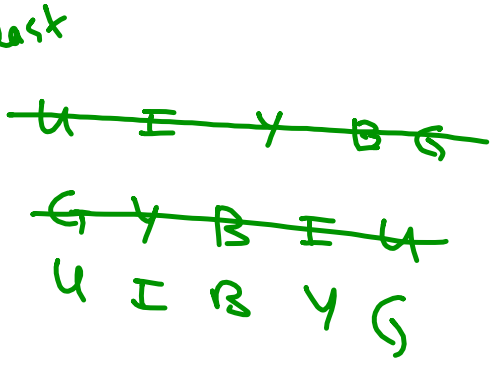
B = blah
I = ick
Y = ynck
U = ugh
G = gloch

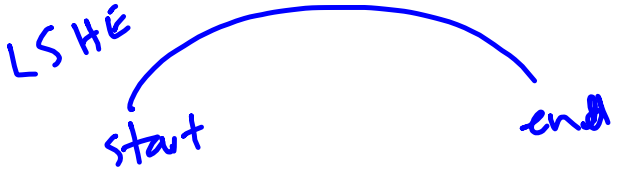
U I B Y G

least

greatest

B not least
Y > U
Y not greatest





H 10
S 5
E 1
L 2

ideal

EL → L (2), L → EL (1)

ES → SL (5), SL → EH (1), EH → (10)

idea 2

SH → H (10), H → SH (5 5)

idea 3

EL → L (2), L → EL (1)

SH → SH (10), SH → EL2 (L 2)

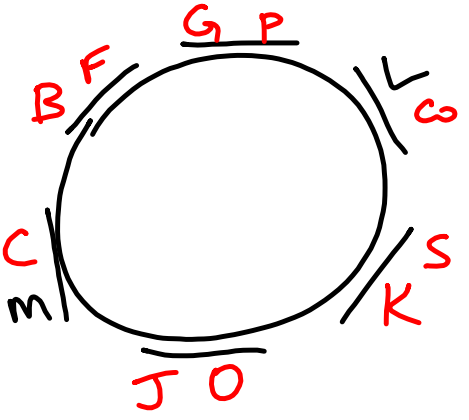
fastest

names

K, L, J, B, C, G

characteristics

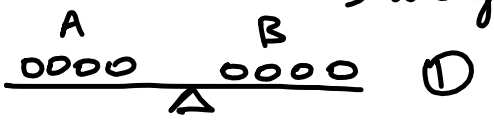
P = pottery
Co = costume
F = foreign exchange
M = music
S = swim
O = owls book



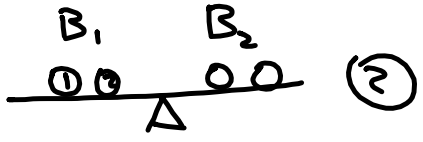
B #11) 8 marbles, 1 is heavier

3 weighings

$8 = 2^3$



side B is heavier:



side B_1 is heavier:

B13) $1.19 quarters, dimes + pennies

(a) can't make change for dollar

✓ 3 q .75

✓ 4 d .40

✓ 4 p .04

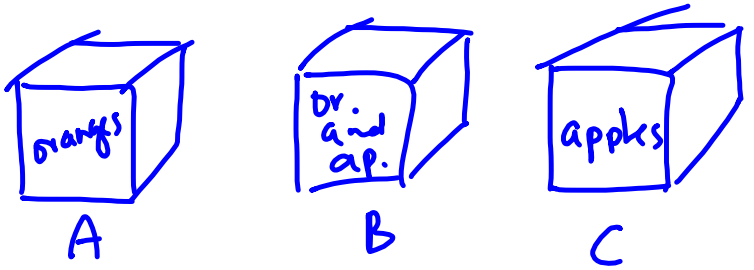
1.19

MC #3) (a) $7 + 6 + 5 + \cdots + 1 = 28$

(d) generalize for n people:

$(n-1)+(n-2)+\cdots+1 = \left(\frac{n-1}{2}\right)(n)$

A13)



doesn't work

X

Pull from box B: if get apple, then B = apples

A = o and a

if I get orange, B = oranges

C = oranges

leaves apples as label ⇒ C = a/o

or a/o as label

A = apples

Quiz (#3)

$5	$10	$20
0	1	2
2	0	2
0	3	1
2	2	1
4	1	1
6	0	1
0	5	0
2	4	0
4	3	0
6	2	0
8	1	0
10	0	0

12 ways

#1)

x = # sandwiches that I could purchase w/ leftover $

$15 - x = 12$

$x = 3$

$(20)\frac{3}{15} = \frac{y}{20}(20)$

$(20)\frac{1}{5} = y$

$y = 4$

$20a = 15b$

a = cost per salad

b = cost per sandwich

$4a = 3b$

$4(4a) = 4(3b)$

$16a = 12b$ ← total cost sandwiches

1.1 A #16) $1+3+5+7+9+\cdots+1001 \overset{?}{=} \frac{501,501}{2} = 250,750.5$

$1+2+3+\cdots+1001 = \frac{1002(1001)}{2} = 501(1001)$

$= 500(1001)+1001$

$= 500,500 + 1,001$

$= 501,501$

$1+3+5+7+\cdots+997+999+1001$

$1+1001 = 1002$

$3+999 = 1002$

$5+997 = 1002$

$\vdots$

$499+503$

501

$1002\left(\frac{499+1}{2}\right)+501$

$= 501(500)+501$

$= 500(500)+1(500)+501$

$= 250,000+500+501$

$= 251,001$

11.A #5)

7 wives
each has 7 sacks
each sack has 7 cats
each cat has 7 kits

living creatures = 1

man wives cats
1 + 7 + 49(7)
+ 7^4
kits