1.2 Explorations with Patterns

Inductive Reasoning--making generalizations based on examples.

Deductive Reasoning--use of axioms, theorems, definitions to prove a statement.

Patterns:

Ex 1: $0^2 = 0$ and $1^2 = 1$ so does $n^2 = n$? false

$n=2, \quad 2^2 \neq 2$

Ex 2: Notice $1 + \underline{2} + 3 = 6$

$9 + \underline{10} + 11 = 30$

$271 + \underline{272} + 273 = 816$

$15 + \underline{16} + 17 = 48$

$20 + \underline{21} + 22 = 63$

Can we see a pattern? And is it true for all whole numbers?

$5 + \underline{6} + 7 = 18$

$12 + \underline{13} + 14 = 39$

$n + (n+1) + (n+2)$
$= 3n+3$
$= 3(n+1)$

$12 + 13 + 14$
$= 13-1 + 13 + 13+1 = 3(13)$

Arithmetic vs. Geometric Sequences

sequence: ordered list of numbers

Arithmetic	Geometric
ex 2, 5, 8, 11, . . . $d = 3$	ex 2, 6, 18, 54, . . . $r=3$
repeatedly add same # (called common difference, d) to get next terms	repeatedly multiply same # (called common ratio, r) to get next terms.

Ex 3: Identify these sequences as arithmetic or geometric. List the next three terms in the sequence. Find the nth term (formula).

(a) $\overset{a_1}{2}, \overset{a_2}{4}, \overset{a_3}{6}, \overset{a_4}{8}$, 10, 12, 14, ... arithmetic $d = 2$

n	a_n
1	$2 = 2(1)$
2	$2+2 = 2(2)$
3	$2+2+2 = 2(3)$
4	$2(4)$
5	$10 = 2(5)$
⋮	
n	$2n$

$a_n = 2n$

(b) $\overset{a_1}{0}, \overset{a_2}{-30}, \overset{a_3}{-60}, \overset{a_4}{-90}$, ... -120, -150, -180, ...

arithmetic, $d = -30$

$a_n = ?$

n	a_n
1	$0 = 0(-30)$
2	$-30 = 1(-30)$
3	$-60 = 2(-30)$
4	$-90 = 3(-30)$
5	$4(-30)$
⋮	
19	$18(-30)$

n	a_n
⋮	
200	$199(-30)$
⋮	
1000	$999(-30)$
⋮	
n	$(n-1)(-30)$

$a_{5000} =$

(c) $\overset{a_1}{5}, \overset{a_2}{10}, \overset{a_3}{20}, \overset{a_4}{40}$, ...

geometric $r = 2$

$a_{11} = 5(2^{11-1}) = 5(2^{10})$
$= 5(1024)$
$= 5120$

n	a_n
1	$5 = 5 = 5(2^0)$
2	$10 = 5(2^1)$
3	$20 = 5(2^2)$
4	$40 = 5(2^3)$
5	$80 = 5(2^4)$
6	$160 = 5(2^5)$
⋮	
n	$\boxed{5(2^{n-1})}$

(d) 4, 16, 64, ...

geometric, $r = 4$

$a_n = 4^n$

n	a_n
1	$4 = 4^1$
2	$16 = 4^2$
3	$64 = 4^3$
⋮	
n	4^n

Ex 4: Joe's annual income has been increasing each year by the same dollar amount. The first year his income was $40,000, and the ninth year, his income was $59,200. In what year was his income $85,600?

d = common diff. arithmetic sequence

a_1

40,000, 40000+d, 40000+2d, 40000+3d, ___, ___, ___, ___, 59200, ___, ___,

a_9

$$59200 = 40000 + 8d$$

$$59200 - 4000 = 8d$$

$$d = \frac{59200-4000}{8} = \$2400$$

$$\frac{59200-40000}{8} = \$2400$$

$$85600 = 40000 + 2400n$$

$$45600 = 2400n$$

$$n = 19$$

in 20th year

Ex 5: Find the first five terms in sequences with the following nth term.

(a) $3^n - 1 = a_n$

n	a_n
1	$3^1 - 1 = 2$
2	$3^2 - 1 = 8$
3	26
4	80
5	242

(b) $4n + 7$

n	a_n
1	11
2	15
3	19
4	23
5	27

(c) $n^3 + 1$

n	$a_n = n^3 + 1$
1	$1^3 + 1 = 2$
2	$2^3 + 1 = 9$
3	28
4	65
5	126

Ex 6: Take a piece of paper and cut the paper into five pieces. Take any one of the pieces and cut it into five pieces, and so on. What sequence can be obtained through this process? What is the total number of pieces after n cuts?

$$1 \xrightarrow{+4} 5 \xrightarrow{+4} 9 \xrightarrow{+4} 13, \ldots$$

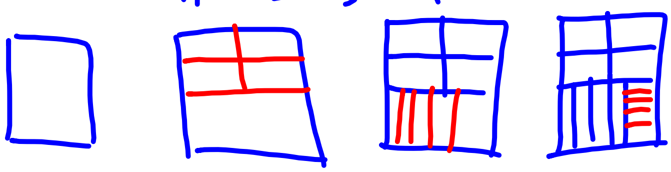
$$a_1,\ a_2,\ a_3,\ a_4$$

arithmetic

$$1+5+9+13+\cdots+\left(n^{th}\ \text{cut}\right)$$

$$=1+5+9+13+\cdots+(1+4(n-1))$$

$$=1+5+9+13+\cdots+(4n-3)$$



n	a_n
1	$1=1+0(4)$
2	$5=1+4$
3	$9=1+2(4)$
4	$13=1+3(4)$
5	$17=1+4(4)$
n	$\cancel{n+4}$ $1+(4)(n-1)=1+4n-4=4n-3$

Ex 7: Find a counter-example for each of these.

(a) If n is a natural number, then (3+n)/3 = n.

try $n=1$ $\quad \frac{3+1}{3}=\frac{4}{3}\neq 1$ $\quad$ our claim is false

(b) If n is a natural number, then $(n-2)^2 = n^2 - 2^2$.

try $n=0$, $(0-2)^2=(-2)^2=4 \neq 0^2-2^2=0-4=-4$

try $n=1$, $(1-2)^2=(-1)^2=1$ but $1^2-2^2=1-4=-3$

$1\neq -3$

$$(n-2)^2=(n-2)(n-2)=n^2-2n-2n+4=n^2-4n+4 \neq n^2-4$$

Ex 8: Tents hold 2, 3, 5, 6 or 12 people. What combinations of tents are possible to sleep 26 people if all tents are fully occupied and only one 12-person tent is used?

2-person	3-person	5-person	6-person	12-person
	1	1	1	1
1			2	1
2		2		1
1	2		1	1
	3	1		1
	1	1	3	
7				1
4			1	1

Ex 9: A student claims that the sequence 4, 4, 4, 4, ... never changes, so it's neither geometric or arithmetic. How do you respond?

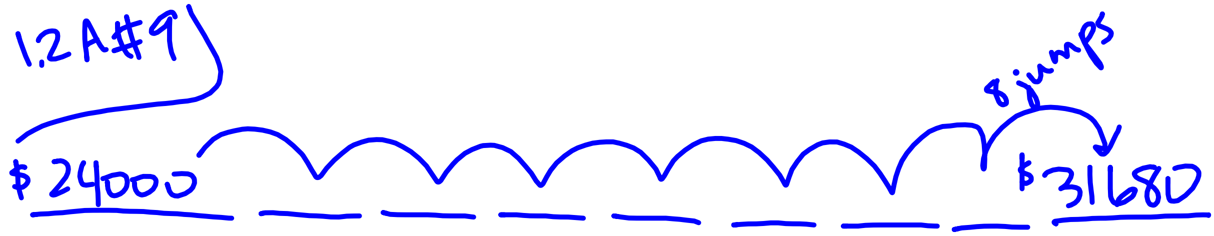
geometric: $r = 1$

arithmetic: $d = 0$

1.2B #8)

day	amt water left
1	15360 L
2	$\frac{1}{2}(15360)$
3	$\frac{1}{2^2}(15360)$
4	$\frac{1}{2^3}(15360)$
⋮	
10	$\frac{1}{2^9}(15360) = \frac{15360}{512} = 30L$

1.2A#9)



—. . . . $45120

$$\frac{31680-24000}{8} = \$960$$

$45120 = 24000 + 960n$

$21120 = 960n$

$22 = n$

year 23