2.1 Numeration Systems

Number--an idea that represents a quantity.

Number uses:

1. to describe how many elements are in a set. (quantity)
2. order (ordinal #) ex I came in 3rd place. (cardinal #)
3. identification (phone #, SSN, uId)

Natural numbers = $\{1, 2, 3, \ldots\}$ "ellipsis" = ...

$\mathbb{N}$

Whole numbers = $\{0, 1, 2, 3, \ldots\}$

$\mathbb{W}$

Historical Numeration Systems:

1. Tally System

Positive Characteristics/Benefits:

- easy (but tedious)

2. Egyptian System

Positive Characteristics/Benefits:

- different symbols for dif. quantities
- additive

Decimal Number	Egyptian Symbol	
1 =		staff
10 =		heel bone
100 =		coil of rope
1000 =		lotus flower
10,000 =		pointing finger
100,000 =		tadpole
1,000,000 =		astonished man

Ex 1 What are the values of these numbers?

(a) 21000+200+30+7 = 21237

(b) 3000+200+40+4 = 3244

3. Roman Numeral System

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Example:
XXXII = 32 XLVIII = 48

40

Positive Characteristics/Benefits:

- few symbols
- additive
- subtractive

ex 439 = CDXXXIX

ex MMCMXCIII
2000 900 90 3
= 2993

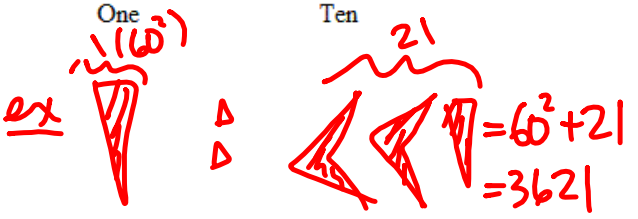
ex CCXLIX = 249
200 40 9

I	1	XXI	21	XLI	41	LXI	61	LXXXI	81
II	2	XXII	22	XLII	42	LXII	62	LXXXII	82
III	3	XXIII	23	XLIII	43	LXIII	63	LXXXIII	83
IV	4	XXIV	24	XLIV	44	LXIV	64	LXXXIV	84
V	5	XXV	25	XLV	45	LXV	65	LXXXV	85
VI	6	XXVI	26	XLVI	46	LXVI	66	LXXXVI	86
VII	7	XXVII	27	XLVII	47	LXVII	67	LXXXVII	87
VIII	8	XXVIII	28	XLVIII	48	LXVIII	68	LXXXVIII	88
IX	9	XXIX	29	XLIX	49	LXIX	69	LXXXIX	89
X	10	XXX	30	L	50	LXX	70	XC	90
XI	11	XXXI	31	LI	51	LXXI	71	XCI	91
XII	12	XXXII	32	LII	52	LXXII	72	XCII	92
XIII	13	XXXIII	33	LIII	53	LXXIII	73	XCIII	93
XIV	14	XXXIV	34	LIV	54	LXXIV	74	XCIV	94
XV	15	XXXV	35	LV	55	LXXV	75	XCV	95
XVI	16	XXXVI	36	LVI	56	LXXVI	76	XCVI	96
XVII	17	XXXVII	37	LVII	57	LXXVII	77	XCVII	97
XVIII	18	XXXVIII	38	LVIII	58	LXXVIII	78	XCVIII	98
XIX	19	XXXIX	39	LIX	59	LXXIX	79	XCIX	99
XX	20	XL	40	LX	60	LXXX	80	C	100
								D	500
								M	1000

4. Babylonian Numeration System

Positive Characteristics/Benefits:

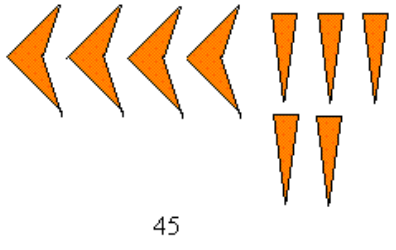
- few symbols
- additive
- has place value
- has "placeholder"



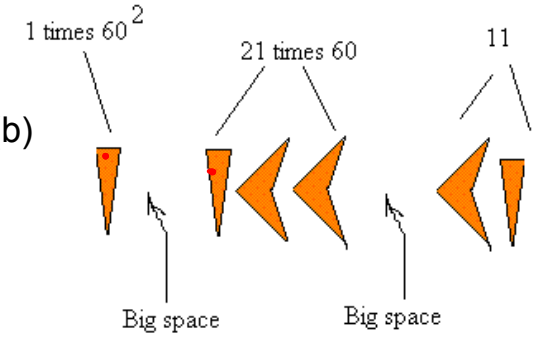
△ was used as a placeholder

Two examples:

(a)



(b)



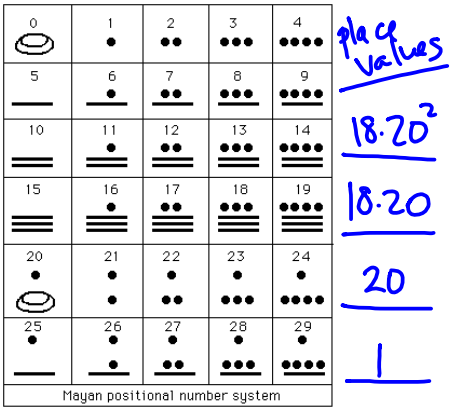
5. Mayan Numeration System

Positive Characteristics/Benefits:

- few symbols
- additive
- has zero
- has place value

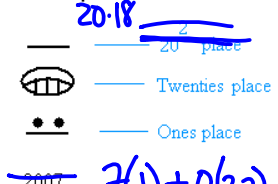
• 1
— 5
0

ex 39



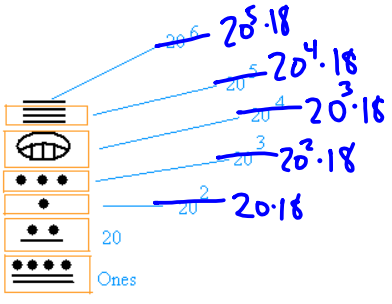
Two examples:

(a)



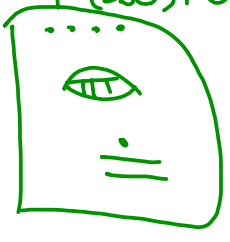
$7(1)+0(20)$
$+5(20 \cdot 18)$
$=7+1800$
$=1807$

(b)



$14(1)+7(20)+1(20 \cdot 18)$
$+3(20^2 \cdot 18)+0(20^3 \cdot 18)$
$+15(20^4 \cdot 18)+5(20^5 \cdot 18)$

ex $1451 = 4(360)+0(20)+11(1)$



1451
-1440
11

7200
360
20
1

ex $3972 = 11(360)+0(20)+12(1)$

6. Hindu-Arabic Numeration System *(our current system)*

Positive Characteristics/Benefits:

- few symbols (0, 1, 2, ..., 9)
- have zero
- additive and multiplicative
- have place value

ex 123

$= 1(100) + 2(10) + 3(1)$

Other Bases

We still have place value!!!

Base 5:

place values:

5^5	5^4	5^3	5^2	5^1	5^0
	625	125	25	5	1

(base 10 numbers)

only allowable digits (symbols) = 0, 1, 2, 3, 4

~~5~~

Ex Convert 12_5 to base 10.

$12_5 = 1(5) + 2(1)$

$= 5 + 2 = 7$

Ex Convert 341_5 to base 10.

$341_5 = 3(25) + 4(5) + 1(1)$

$= 75 + 20 + 1 = 96$

Ex Convert 39 to base 5.

$39 = 1(25) + 2(5) + 4(1)$

$= 124_5$

$39 - 25 = 14$

$14 - 10 = 4$

Ex Convert 401 to base 5.

$= 3101_5$

125	125	5	1
•	•		•

$401 = 3(125) + 1(25) + 0(5) + 1(1)$

$0^0 = ?$

$3^0 = 1$ $\quad 0^3 = 0$

$2^0 = 1$ $\quad 0^2 = 0$

$1^0 = 1$ $\quad 0^1 = 0$

$0^0 = 1?$ $\quad 0^0 = 0?$

0^0 is undefined

$5^3 = 125$ ÷5

$5^2 = 25$ ÷5

$5^1 = 5$

$5^0 = 1$

Convert these numbers to base 10.

(a) 1011101_2 128 64 32 16 8 4 2 1 base 2

$= 1(64)+0(32)+1(16)+1(8)+1(4)+0(2)+1(1)$

$= 93$

(b) 1237_8 $= 672$ 512 64 8 1

$1237_8 = 1(512)+2(64)+3(8)+7(1)$

$= 512+128+24+7 = 671$

(c) $2ET_{12} = 2(12^2)+E(12)+T(1) = 2(144)+11(12)+10(1)$

$= 288+132+10 = 430$

(d) $654_7 = 6(7^2)+5(7)+4$

$= 6(49)+35+4$

$= 294+39$

$= 333$

Convert these numbers from base 10 to the indicated base.

(a) 76 to base 2

128 64 32 16 8 4 2 1

$= 1(64) + 0(32) + 0(16) + 1(8) + 1(4) + 0(2) + 0(1)$

$= 1001100_2$

$76 - 64 = 12$ $\quad 12 - 8 = 4$

(b) 982 to base 4 $= 33112_4$

256 64 16 4 1

$982 = 3(256) + 3(64) + 1(16) + 1(4) + 2(1)$

$982 - 768 = 214$ $\quad 214 - 192 = 22$ $\quad 22 - 16 = 6$

(c) 131 to base 3 $= 11212_3$

243 81 27 9 3 1

$131 = 1(81) + 1(27) + 2(9) + 1(3) + 2(1)$

$131 - 81 = 50$ $\quad 50 - 27 = 23$ $\quad 23 - 18 = 5$

(d) 519 to base 8

512 64 8 1

$= 1(512) + 0(64) + 0(8) + 7(1)$

$= \boxed{1007_8}$

2.1 HW Questions

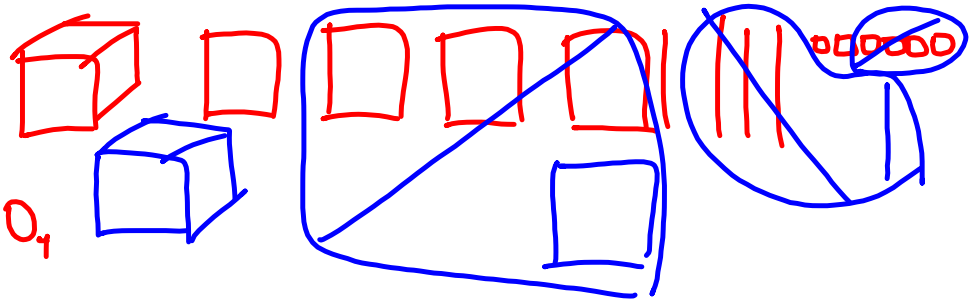
A#10)

base 4

| = 4 or 10_4

unit | long flat | cube

base blocks

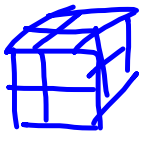


211_4

A#13) $2032_4 = 2(4^3) + 0(4^2) + 3(4) + 2(1)$

$= 2(64) + 12 + 2 = 142$

B#18) 1001_2



B#15)

(a) 66_7 100_7 101_7

(b) 1111_2 10000_2 1001_2

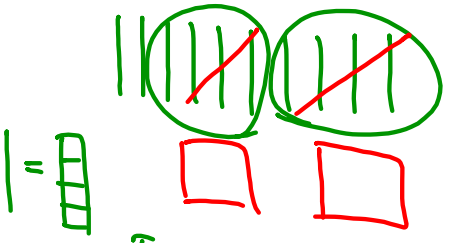
(c) 100_2 101_2 110_2

A#4) (a) base 2, biggest 3-digit # 111_2

(b) base 12, T=ten, E=eleven EEE_{12}

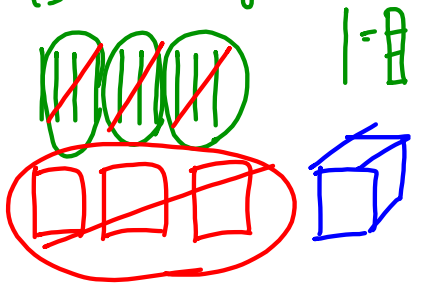
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E

B#21) (a) 10 longs in base 4



I need 4 total
(2 flats and 2 longs)

(b) 10 longs in base 3



I need 2 total:
1 cube and 1 long

2.1 B#23) (a) 432_6

$=4(6^2)+3(6)+2(1)$

(b) 1101_2

$=1(2^4)+1(2^3)+0(2^2)+1(2)+1(1)=16+8+2+1$
$=27$

(c) $E29_{12}$

$=11(12^2)+2(12)+9(1)$

B#25) 1 cup, 1 pt, 1 qt

1 pt = 2 cups

1 qt = 2^2 cups

111_2

1.2 MC #11) $2(9+8+7+6+5+4+3+2+1)$

$=2(45)=90$

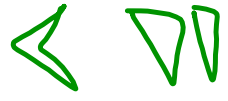
2.1A #5)

(a) 72 to Mayan #

$72 = 3(20) + 12(1)$

360
20
1

ex $2(360) + 0(20) + 15(1)$

(b) 

$10(60) + 2(1) = 602$

$602 = 1(360) + 12(20) + 2(1)$

602
-360
242

Babylonian

▽ = 1

< = 10

△△ = place-holder

place values:

60^3 60^2 60 1

$60^2 = 3600$

$60^3 = 216{,}000$

Mayan

• = 1

— = 5

 = 0

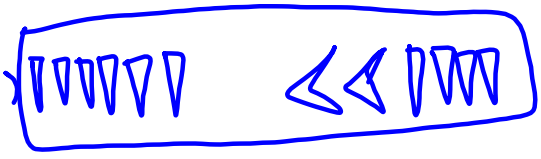
place values: 7200

360

20

1

Quiz 3

1) $384 = 6(60) + 24(1)$ 

2) $384 = 1(360) + 1(20) + 4(1)$

⋮

....

3) $453_6 = 4(36) + 5(6) + 3(1)$

$= 144 + 30 + 3 = 177$

4) 89 to base 3 243 81 27 9 3 1

$89 = 1(81) + 0(27) + 0(9) + 2(3) + 2(1)$

$= 10022_3$

EC: $W = \{0, 1, 2, 3, \ldots\}$