3.1 & 3.2 Whole Number Addition and Subtraction

Addition & Subtraction--binary operations

require two #s to act on ex $3+5$

~~3+~~

$W = \{0, 1, 2, 3, \ldots\}$

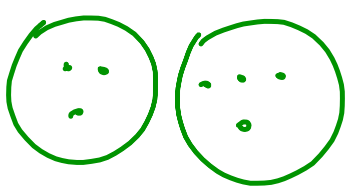
Properties of Addition (with Whole numbers): "Whole # are closed under additn"

1. Closure-- if I add any 2 whole #s, I get back a whole #
2. Commutativity-- order doesn't matter; $a+b=b+a$
3. Associativity-- grouping doesn't matter; $(a+b)+c=a+(b+c)$
4. Additive Identity-- 0 $a+0=0+a=a$

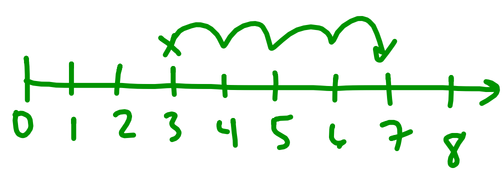
(0 preserves the identity of any # it's added to)

Set Model

ex
$3+4 = 7$



Measurement Model



$3+4=7$

Addition Thinking Strategies:

1. Doubles $7+7+6=2(7)+6=14+6=20$
2. Add zero $19+23=19+1+23-1=20+22=42$
3. Commutativity/associativity $7+5+4+21=7+(4+5)+21$ $=7+9+21=7+30=37$
4. Counting by 2s or 5s $13+8=21$
5. Doubles +/- 1 $7+6=(6+1)+6=13$
6. Grouping by tens
7. Counting on $19+11+3+5+7=30+10+5=45$

(most common for finger counting) $6+5=11$

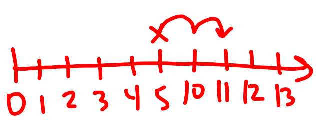
Ex Find three different ways to add:

5 + 9
① $5-1+9+1=4+10=14$
② $4+(1+9)=4+10=14$
③ counting on
9 ⌒⌒⌒⌒⌒ 14

14 + 28 + 36
① $14+36+28=50+28=78$
② $10+4+6+30+28=50+28=78$
③ $15+35+28=78$

51 + 89
① $30+90=140$
② $50+80+1+9=130+10=140$
③ $50+90$ then count by tens

$5_6+2_6=11_6$
①



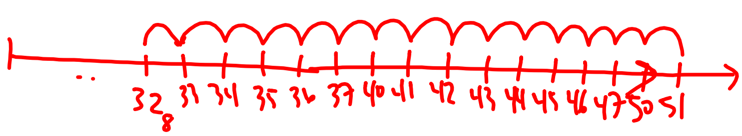
② $5_6+1_6+1_6=10_6+1_6=11_6$

17_8+32_8

base 8 +	1	2	3	4	5	6	7
1	2	3	4	5	6	7	10
2	3	4	5	6	7	10	11
3	4	5	6	7	10	11	12
4	5	6	7	10	11	12	13
5	6	7	10	11	12	13	14
6	7	10	11	12	13	14	15
7	10	11	12	13	14	15	16

① $17_8+32_8=$
$=10_8+7_8+30_8+2_8$
$=10_8+7_8+2_8+30_8$
$=10_8+11_8+30_8=51_8$

② $17_8+32_8=$
$10_8+30_8+7_8+2_8=$
$40_8+11_8=51_8$



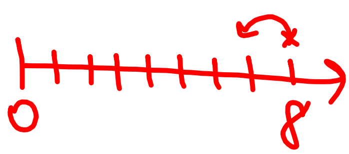
Subtraction

Take-away approach

(use w/ set or measurement model)

$5-3=2$

$8-1=7$



Missing addend approach

ex $\$20 - \$3.84 = ?$

$\Longleftrightarrow \$3.84 + ? = \$20$

ex $15 - 11 = ?$

$\Longleftrightarrow 11 + ? = 15$

Four-fact families:

$8-5=3$ $3+5=8$

$8-3=5$ $5+3=8$

TABLE 4.1. A Taxonomy of Addition and Subtraction Word Problems

CHANGE-ADD-TO with	. . . UNKNOWN OUTCOME	. . . UNKNOWN CHANGE	. . . UNKNOWN START
	Alexi had 5 candies. Barb gave him 3 more. How many candies does he have altogether now?	Alexi had 5 candies. Barb gave him some more. Now he has 8 altogether. How many candies did Barb give him?	Alexi had some candies. Barb gave him 3 more. Now he has 8 altogether. How many candies did he start with?
CHANGE-TAKE-AWAY with	. . . UNKNOWN OUTCOME	. . . UNKNOWN CHANGE	. . . UNKNOWN START
	Alexi had 8 candies. He gave 5 to Barb. How many candies does he have left?	Alexi had 8 candies. He gave some to Barb. Now he has 3 left. How many candies did he give to Barb?	Alexi had some candies. He gave 5 to Barb. Now he has 3 left. How many candies did he start with?
PART-PART-WHOLE with	. . . UNKNOWN WHOLE	. . . UNKNOWN SECOND PART	. . . UNKNOWN FIRST PART
	Alexi had 5 fireballs and 3 lollipops. How much candy did he have altogether?	Alexi had 5 fireballs and some lollipops. He had 8 candies altogether. How many were lollipops?	Alexi had some fireballs and 3 lollipops. He had 8 candies altogether. How many were lollipops?
EQUALIZE with	. . . UNKNOWN DIFFERENCE	. . . UNKNOWN SECOND PART	. . . UNKNOWN FIRST PART
	Alexi had 8 candies. Barb had 5. How many more does Barb have to buy to have as many as Alexi?	Alexi had 8 candies. Barb had to get 3 more candies to have the same number as Alexi. How many candies did Barb start with?	Alexi had some candies. Barb, who had 5 candies, had to get 3 more to have the same number as Alexi. How many candies did Alexi have?
COMPARE with	. . . UNKNOWN DIFFERENCE	. . . UNKNOWN SECOND PART	. . . UNKNOWN FIRST PART
	Alexi had 8 candies. Barb had 5. How many more candies did Alexi have than Barb?	Alexi had 8 candies. He had 3 more than Barb. How many candies did Barb have?	Alexi had some candies. He had 3 more than Barb who had 5. How many candies did Alexi have?

Note. The examples shown above for EQUALIZE and COMPARE problems are the "more" versions. "Less" versions could also be written for each. For example, the less version of the EQUALIZE with UNKNOWN DIFFERENCE would read: Alexi had 8 candies. Barb had 5. How many does Alexi have to give up to have as many as Barb?

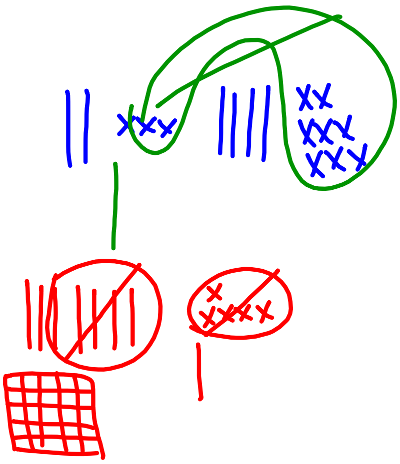
Algorithm-- series of steps to follow; "a recipe for a math problem"

Addition

(a) base pieces

$23 + 48 = 71$

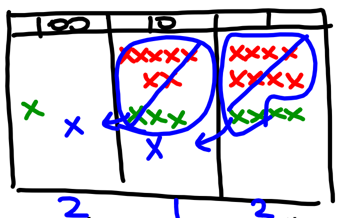
$31_5 + 44_5 = 130_5$



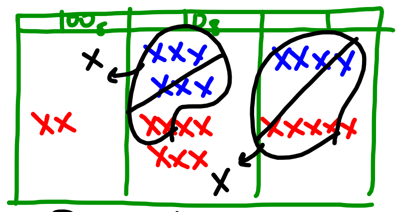
$(1 = 5 \text{ or } 10_5)$

(b) chip abacus

$78 + 134 = 212$



$64_8 + 275_8 = 361_8$



3 6 1

(c) place-value representation

	100	10	1
	1	6	7
+	4	2	9
	5	8	~~16~~
		1	6
	5	9	6

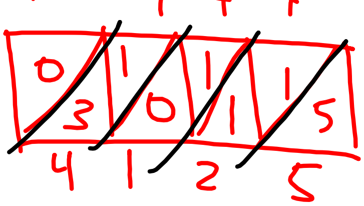
(d) intermediate algorithm

$384 + 192 = 576$

$$\begin{array}{r} 384 \\ +\,192 \\ \hline 006 \\ 170 \\ 400 \\ \hline 576 \end{array}$$

(e) lattice method

$$\begin{array}{r} 3148 \\ +\;\;977 \end{array}$$



(f) standard algorithm

(1)

$$\begin{array}{r} \overset{1}{3}\overset{1}{7}41 \\ +\;498 \\ \hline 4239 \end{array}$$

(2)

$$\begin{array}{r} \overset{1}{3}401_5 \\ +\;333_5 \\ \hline 4234_5 \end{array}$$

Subtraction

(a) base pieces

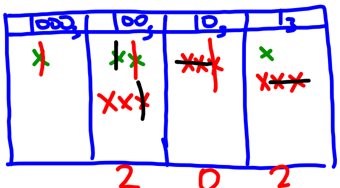
$$\begin{array}{r} 714_9 \\ -\ 87_9 \\ \hline 616_9 \end{array}$$

(e) standard algorithm

$$\begin{array}{r} 100012 \\ -\ 9674 \\ \hline 90338 \end{array}$$

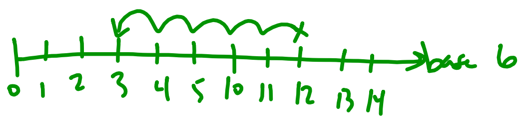
(b) chip abacus

$1201_3 - 222_3 = 202_3$



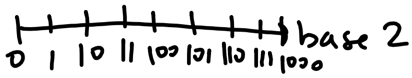
(c) place-value representation

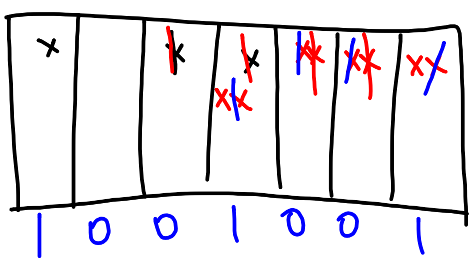
$$\begin{array}{r} 425_6 \\ -\ 354_6 \\ \hline 31 \end{array}$$



(d) intermediate algorithm

$$\begin{array}{r} 1011000_2 \\ -\ 1111_2 \\ \hline 01 \\ 00 \\ 00 \\ 01 \\ 100 \\ \hline 1001001_2 \end{array}$$



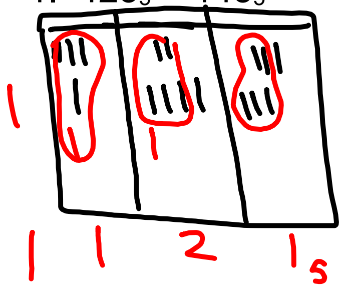


$$\begin{array}{r} 3201 \\ -\ 769 \\ \hline -8 \\ -60 \\ -500 \\ +\ 3000 \\ \hline 2432 \end{array}$$

$$\begin{array}{r} 3201 \\ -\ 769 \\ \hline 2432 \end{array}$$

More examples:

1. $423_5 + 143_5$

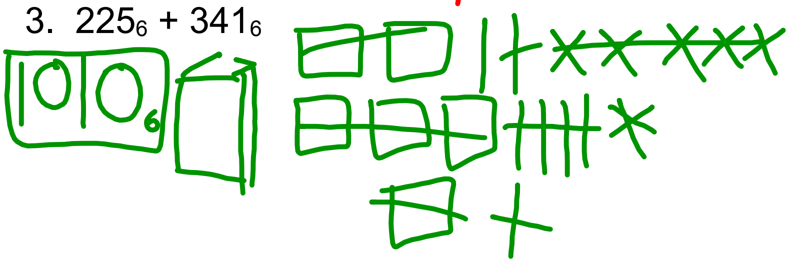


2. $301_7 - 265_7$

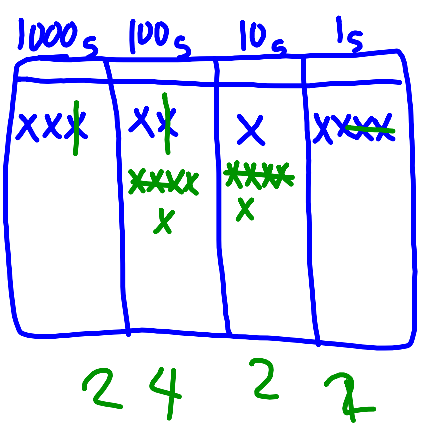
$11_7 = 8_{10}$

$$\begin{array}{r} 2\,\cancel{3}\,\cancel{0}\,\cancel{1}_7 \\ -\;265_7 \\ \hline 003_7 \end{array}$$

3. $225_6 + 341_6$

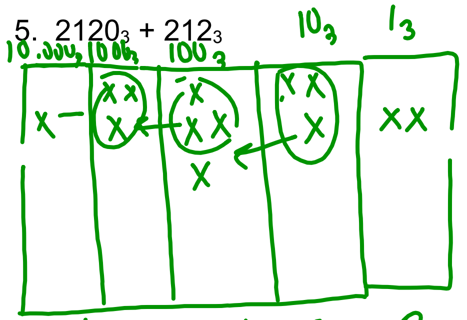


4. $3214_5 - 242_5$



2422_5

5. $2120_3 + 212_3$



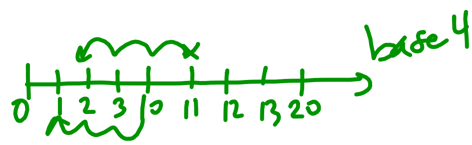
$1\ 0\ 1\ 0\ 2_3$

6. $12210_3 - 201_3$

$$\begin{array}{r} 12210_3 \\ -\quad 201_3 \\ \hline 12002_3 \end{array}$$

7. $3112_4 - 331_4$

$$\begin{array}{r} 3112_4 \\ -\quad 331_4 \\ \hline 2121_4 \end{array}$$



8. $101010001_2 + 111111_2$

$$\begin{array}{r} 101010001_2 \\ +\quad 111111_2 \\ \hline \end{array}$$

110010000_2

What are these kids thinking?

$$\begin{array}{r} 23 \\ -15 \\ \hline 12 \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}\overset{15}{\cancel{6}}2 \\ -237 \\ \hline 325 \end{array}$$

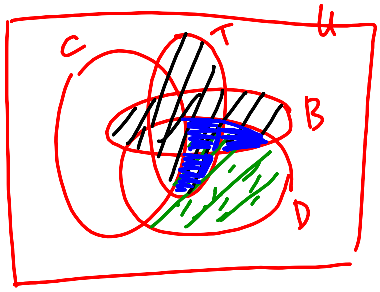
$$\begin{array}{r} \overset{3}{\cancel{5}}{}^{1}6{}^{1}2 \\ -287 \\ \hline 185 \end{array}$$

$$\begin{array}{r} 25 \\ +37 \\ \hline 125 \end{array}$$

$$\begin{array}{r} 25 \\ +37 \\ \hline 53 \end{array}$$

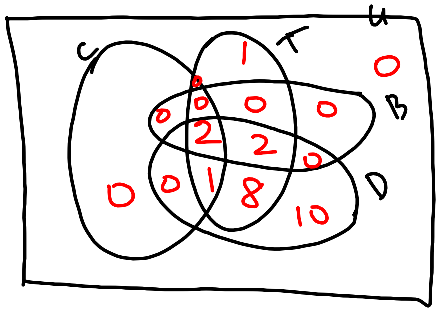
~~Hwset 4 VD wkshts~~

7) shade $(T\cup B)\cap(D-C)$



T= trax
C= carpool
D= drive own car
B= bus

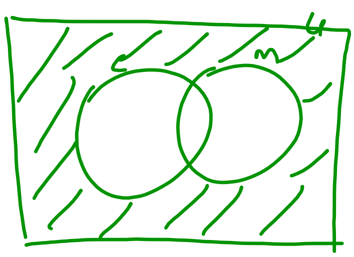
1)



2) {Kelly}

√8 T ∩ D only
√10 D only
√1 T only
√2 all 4
√2 T ∩ B ∩ D
√1 T ∩ C ∩ D

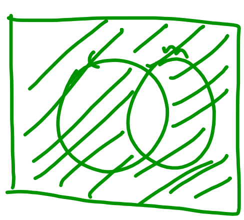
4)



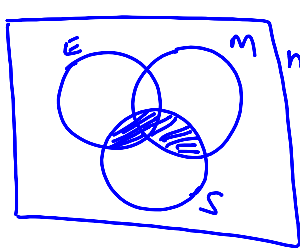
$\bar{C}\cap\bar{M}$

$n(\bar{C}\cap\bar{M}) = 2$

$\overline{C\cap M}$



5)



$n(S\cap(E\cup M)) = 6$

3.1 HW

A5) (a) $B = \{0\}$ yes

(b) $T = \{3n \mid n \in \mathbb{W}\} = \{0, 3, 6, 9, \ldots\}$ yes

(d) $V = \{3, 5, 7\}$ no

$3+5=8 \notin V$

(f) $C = \{0, 1\}$ no

$1+1=2 \notin C$

A6) A is closed under additn and contains 2 and 3

$A = \{2, 3, 4, 5, 6, 7, 8, 9, \ldots\}$

A10) a < b when a can be subtracted from b, and still give back a whole #.

B5) (b) $T = \{0, 4, 8, 12, 16, \ldots\}$ yes $4n + 4m = 4(n+m)$

(c) $F = \{5, 6, 7, 8, 9, \ldots\}$ yes

3.1 A

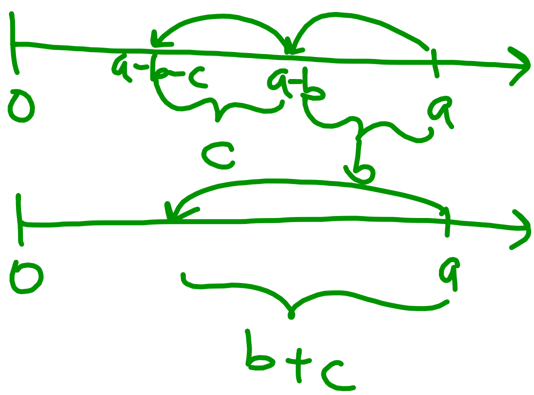
19) (e) $x+8 = 5 + (x+3)$

$$x+8 = x+8$$
$$-8 \qquad -8$$
$$x = x \qquad x \in W$$

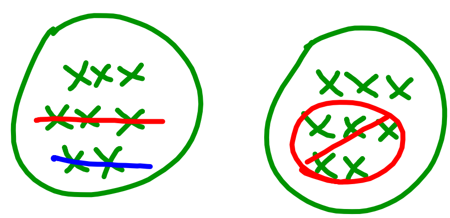
(g) $x-3 = x+1$

$$-x \qquad -x$$
$$-3 \neq 1 \Rightarrow \text{N.S.}$$

3.1 MC #4) (b) $a-b-c = a-(b+c)$



ex $8-3-2 = 8-(3+2)$



3 3

MC #18)

$$\left.\begin{array}{l} 8-5=3 \\ 5-2=3 \\ 6-1=5 \\ 12-7=5 \end{array}\right\}$$ claim: W closed under subtraction

$8-9 = -1 \notin W$

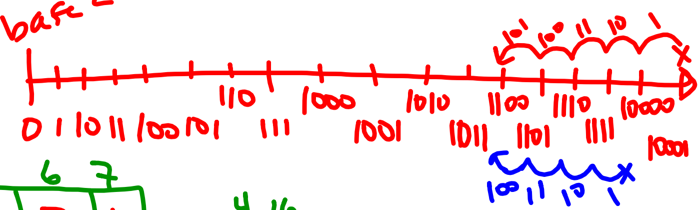
3.2 A16)

$$\begin{array}{r} \overset{1}{4}\overset{1}{3}2 \\ \cancel{9}\cancel{7}6 \\ +\ 1418 \\ \hline 2826 \end{array}$$

B#12f)

$$\begin{array}{r} 10001_2 \\ -\ 101_2 \\ \hline \end{array}$$

base 2



A13)

+	1	2	3	4	5	6	7
1	2	3	4	5	6	7	10
2	3	4	5	6	7	10	11
3	4	5	6	7	10	11	12
4	5	6	7	10	11	12	13
5	6	7	10	11	12	13	14
6	7	10	11	12	13	14	15
7	10	11	12	13	14	15	16

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{16}{\cancel{7}}\,\overset{10}{3} \\ -\ 77_8 \\ \hline 474_8 \end{array}$$

B18) (c) $E08_{12} - \_\_\_\_ = 9_{12}$

four-fact families

ex

$2+4=6$

$4+2=6$

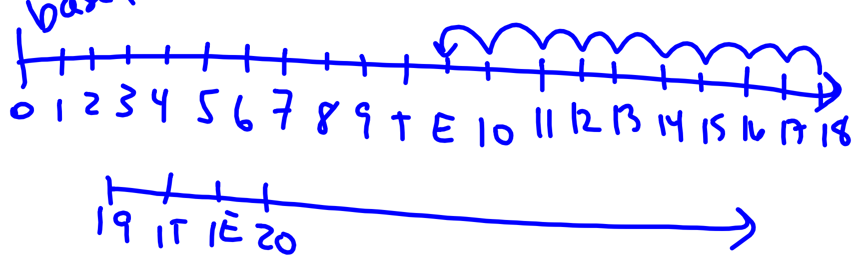
$6-4=2$

$6-2=4$

$E08_{12} - 9_{12} = \underline{TEE_{12}}$

$$\begin{array}{r} \overset{T}{\cancel{E}}\,\overset{E}{\cancel{0}}\,8_{12} \\ -\ 9_{12} \\ \hline TEE \end{array}$$

base 12

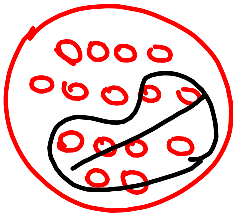


(d) $100_2 + \_\_\_\_ = 10000_2$

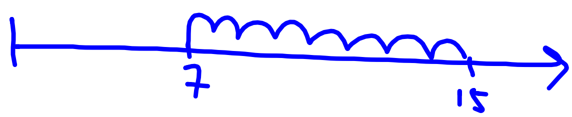
$10000_2 - 100_2 = \underline{1100_2}$

2) $15 - 8 = 7$

15 cookies and 8 children. Each child gets one cookie. How many cookies are left?



Harry is 15 miles away from SLC. He drove another 8 miles. How many miles are left?



3) $$\begin{array}{r} 513 \\ -311 \end{array}$$

$5 - 3 = 2$

$13 - 11 = 2$

$\Rightarrow 513 - 311 = 202$

5) (a) $$\begin{array}{r} \overset{2}{\cancel{3}}\overset{4}{\cancel{0}}{}^{1}12_5 \\ -\ 322_5 \\ \hline 2140_5 \end{array}$$

(b) $$\begin{array}{r} \overset{T}{\cancel{E}}\overset{E}{\cancel{0}}{}^{1}8_{12} \\ -\ 9_{12} \\ \hline TEE \end{array}$$

(c) $$\begin{array}{r} 100001_2 \\ +\ 11010_2 \\ \hline 111011_2 \end{array}$$

(d) $$\begin{array}{r} {}^{1}3\,{}^{1}3\,{}^{1}2\,5_6 \\ +\ 345_6 \\ \hline 4114_6 \end{array}$$