3.3 & 3.4 Whole Number Multiplication and Division

Multiplication & Division--binary operations (need 2 #s)

ex $8 \div 4$

Properties of Multiplication (with Whole numbers):

1. Closure-- set of whole #s is closed under mult.
2. Commutativity-- ex $3 \cdot 4 = 4 \cdot 3$ (order doesn't matter) $ab = ba$
3. Associativity-- (grouping doesn't matter) ex $3(5 \cdot 2) = (3 \cdot 5)2$ $(ab)c = a(bc)$
4. Multiplicative Identity = 1 $a \cdot 1 = 1 \cdot a = a$
5. Distributivity-- through addition/subtraction $a(b \pm c) = ab \pm ac$ ex $3(2+5) = 3(2) + 3(5)$
6. Multiplication Property of Zero--

if $a \cdot b = 0$, then $a = 0$ or $b = 0$.

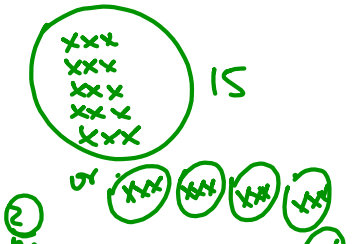
ex $(x-5)(x+3) = 0$

$x = 5, -3$

Multiplication Approaches:

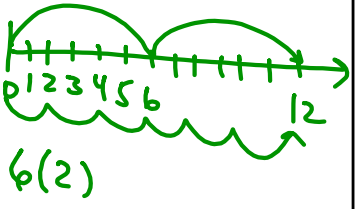
Repeated Addition

1) Set 5(3)



2) Measurement

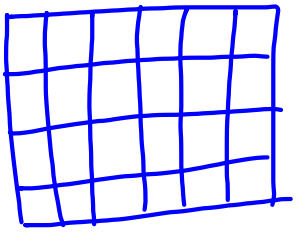
2(6)



6(2)

Rectangular Array

$4 \cdot 6$ (area model)



Cartesian Product

$A = \{a, b\}$

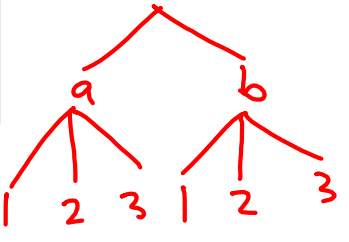
$B = \{1, 2, 3\}$

$A \times B$ read "A cross B"

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$n(A \times B) = n(A) \cdot n(B)$

$6 = 2(3)$ ✓



Ex Use mental math strategies and the multiplication properties to simplify these expressions.

$31(74) + 39(74) = (31+39)(74) = 70(74) = 70(70+4)$

$= 4900 + 280$

$= 5180$

$25(90) = (25(9))10$

$= 225(10) = 2250$

or $25(100-10)$

$= 2500 - 250 = 2250$

$47(9) = 40(9) + 7(9) = 360 + 63 = 423$

$20_3(11_3) = 20_3(1_3) + 20_3(10_3) = 20_3 + 200_3 = 220_3$

$41_7(6_7) = 40_7(6_7) + 1_7(6_7)$

$= 330_7 + 6_7$

$= 336_7$

$6_7 \cdot 2_7 = 15_7$

$6_7 \cdot 4_7 = (6_7 \cdot 2_7)2_7$

$= 15_7\, 2_7 = 33_7$

Division

Partitive

answers the qn:
"How many are in each group?"

Measurement

answers the qn:
"How many groups?"

Ex: Classify each of the following division problems as examples of either partitive or measurement division.

(a) A certain airplane climbs at a rate of 300 feet per second. At this rate, how long will it take the plane to reach a cruising altitude of 27,000 feet?

$27000 \div 300$ measurement

(b) A group of 15 friends pooled equal amounts of money to buy lottery tickets for a $1,987,005 jackpot. If they win, how much should each friend receive?

partitive

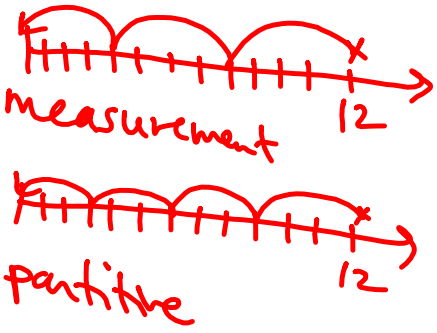
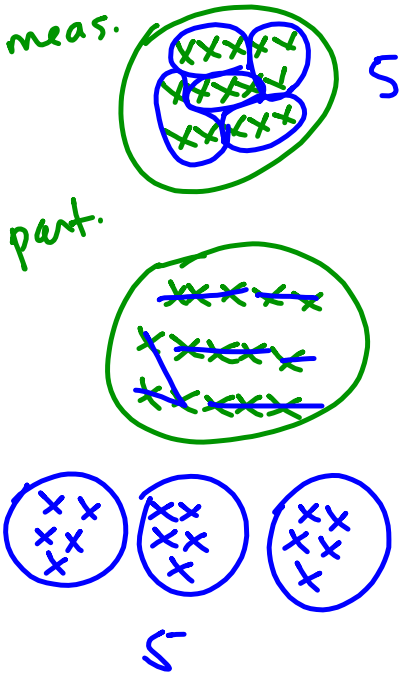
$1,987,005 \div 15$

(c) Shauna baked 54 cookies to give to her friends. She wants to give each friend a plate with 6 cookies on it. How many friends can she give cookies to?

measurement

$54 \div 6$

Division Approaches:

Repeated Subtraction	Set Model	Missing Factor Model
ex 12 ÷ 4  	ex 15 ÷ 3  	18 ÷ 3 = ? ⟺ 18 = 3 · ? ⇒ ? = 6

The Division Algorithm:

Given any whole numbers a and b with (b not equal to 0), there exist whole numbers q (quotient) and r (remainder) such that

$a = bq + r$ with $0 \leqq r < b$.

$a \div b = q + \frac{r}{b}$

$27 \div 5 = ?$

ex $27 = 5(5) + 2$ $\quad 27 \div 5 = 5 + \frac{2}{5}$

(Vocabulary: When a is divided by b and the remainder is zero, then we can say "a is divisible by b" or "b is a divisor of a" or "b divides a.")

Ex: 69 ÷ 9

or $\frac{69}{9} = 7\frac{6}{9}$

$69 = q(9) + r = 7(9) + 6$

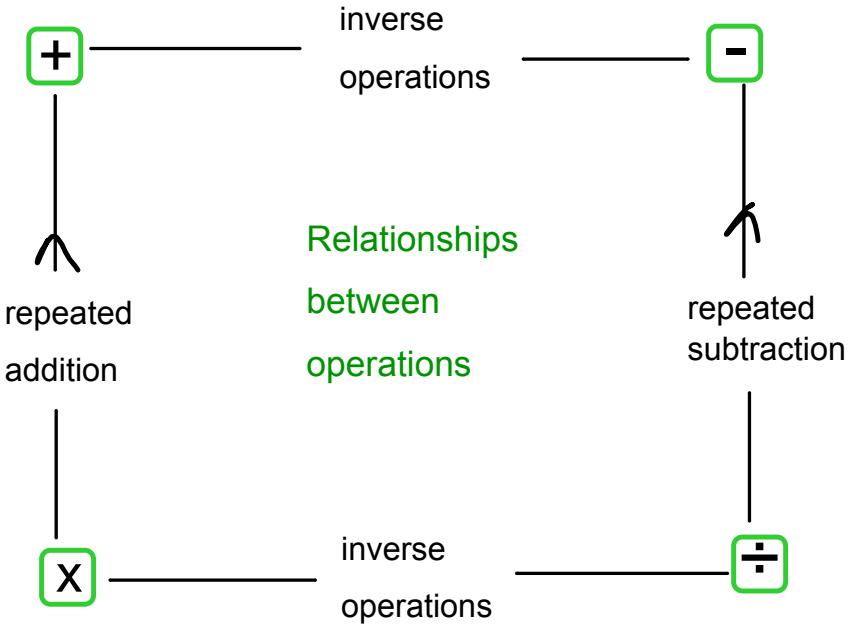
Ex. When the marching band was placed in rows of 5, one member was left over. When the members were placed in rows of 6, there was still one member left over. However, when they were placed in rows of 7, nobody was left over. What is the smallest number of members in the band?

91 $\quad a = bq + r$

$30m + 1 = 7n$

m	30m+1	divisible by 7?
1	31	no
2	61	no
3	91	13 yes

Inverse Operations:



Four-Fact Families:

① Use 3x8=24

② 8x3=24

③ 24÷3=8

④ 24÷8=3

Division by zero is undefined!!

$\frac{5}{0}$ is undefined

"5 cookies to give to 0 friends."
It's ill-defined.

$\frac{0}{5} = ?$

$\Rightarrow$ answer is 0

"have zero cookies to divide among my 5 friends. How many cookies does each friend get?"

① $\frac{5}{0} = ? \iff 5 = ? \cdot 0$

no good answer
answer is undefined

"undefined because nothing works"

② $\frac{0}{0} = ? \iff 0 = ? \cdot 0$

"undefined because everything works"

Order of Operations Reminder:

P arantheses

E xponents

MD multiplication/division

AS addition/subtraction

ex $1+30\div6\cdot3$

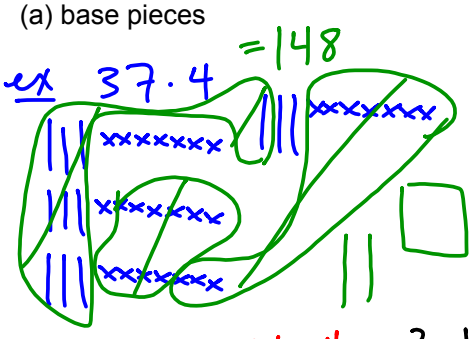
$=1+5\cdot3$

$=1+15=16$

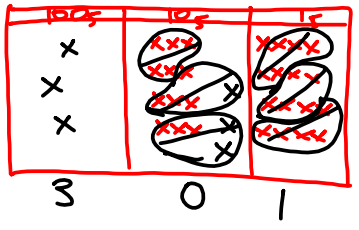
Multiplication

(a) base pieces

ex $37 \cdot 4$ $= 148$



(b) chip abacus $34_5 \cdot 4_5 = 301_5$



(c) horizontal format

ex $349(70)$

$= (300+40+9)(70) = 300(70) + 40(70) + 9(70)$

$= 21000 + 2800 + 630$

$= 23{,}800 + 630 = 24{,}430$

(d) intermediate algorithm

ex

$$\begin{array}{r} 724 \\ \times\ 51 \\ \hline 724 \\ 200 \\ 1000 \\ 35000 \\ \hline 36{,}924 \end{array}$$

$724(51) = (700+20+4)(50+1)$

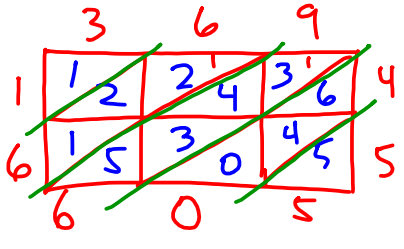
$= 700(50) + 20(50) + 4(50)$

$+ 724(1)$

(e) lattice method

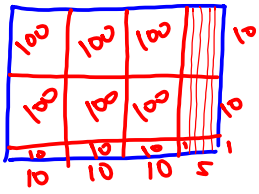
ex

$$\begin{array}{r} 369 \\ \times\ 45 \\ \hline 16{,}605 \end{array}$$



(f) area model

ex $35(21) = 6(100) + 13(10) + 5(1) = 735$



(g) standard algorithm

ex

$$\begin{array}{r} {}^{2}207 \\ \times\ 34 \\ \hline 828 \\ 6210 \\ \hline 7028 \end{array}$$

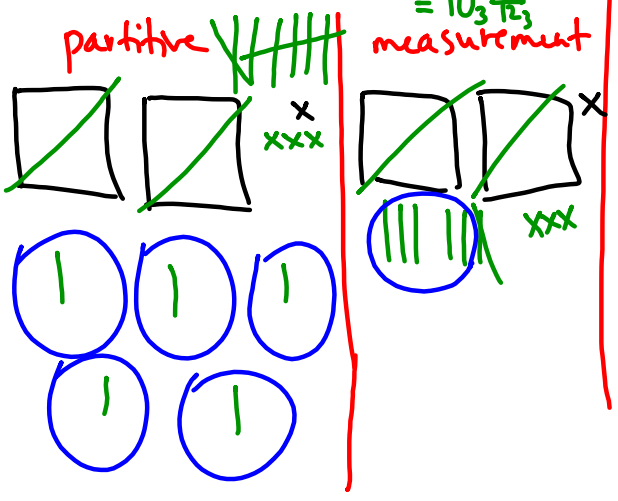
Division

(a) base pieces

ex $201_3 \div 12_3 = 10_3 \text{ R } 11_3$

$= 10_3 \frac{11_3}{12_3}$

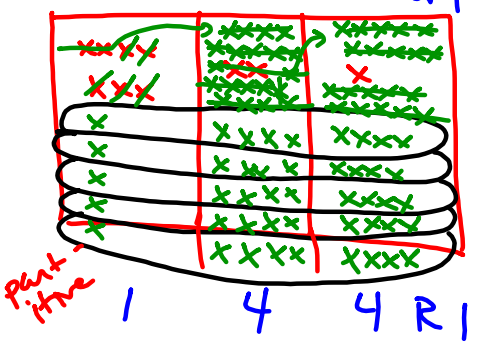
partitive measurement



(b) chip abacus

ex $721 \div 5 = 144 \text{ R } 1$

$\approx 144\frac{1}{5}$

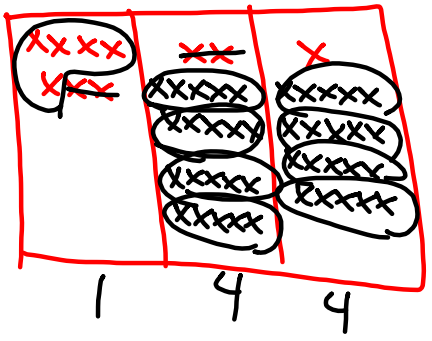


partitive

1 4 4 R 1

$721 \div 5 = 144 \text{ R } 1$

(measurement)



1 4 4

(c) scaffolding method

ex

$$\begin{array}{r} 159 \text{ R } 18 \\ 23\overline{)3675} \\ -2300 \quad 100 \\ \hline 1375 \\ -460 \quad 20 \\ \hline 915 \\ -460 \quad 20 \\ \hline 455 \\ -230 \quad 10 \\ \hline 225 \\ -207 \quad 9 \\ \hline 18 \end{array}$$

(d) intermediate algorithm

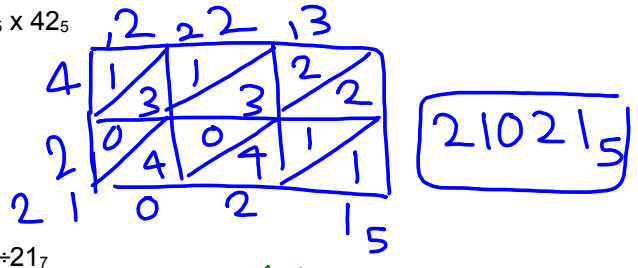
ex

$$\begin{array}{r} 7 \\ 30 \\ 100 \\ \hline 51\overline{)7008} \\ -5100 \\ \hline 1908 \\ -1530 \\ \hline 378 \\ -357 \\ \hline 21 \end{array}$$

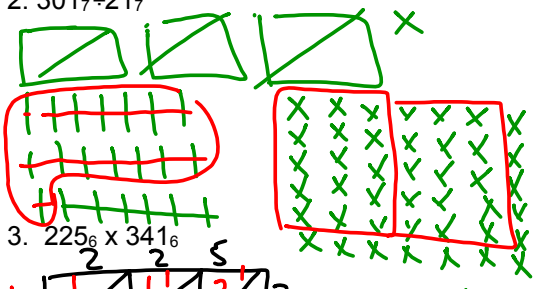
$7008 \div 51 = 137 \text{ R } 21$

More examples:

1. $223_5 \times 42_5$



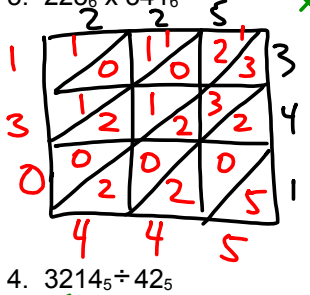
2. $301_7 \div 21_7$



$12_7 R\ 16_7$

or $12_7 \frac{16_7}{21_7}$

3. $225_6 \times 341_6$



4. $3214_5 \div 42_5$

base 5

$$\begin{array}{r} 34 R 31_5 \\ 42 \overline{)3214} \\ -420 \quad 10 \\ \hline 244 \\ -420 \quad 10 \\ \hline 324 \\ -420 \quad 10 \\ \hline 204 \\ -42 \quad 1 \\ \hline 112 \\ -42 \quad 1 \\ \hline 70 \\ -42 \quad 1 \\ \hline 23 \\ -42 \quad 1 \\ \hline 31 \end{array}$$

$$\begin{array}{r} 12_7 R 16_7 \\ 21 \overline{)301} \\ -210 \quad 10_7 \\ \hline 61 \\ -42 \quad 2 \\ \hline 16 \end{array}$$

5. $12210_3 \div 201_3$

$22_3 R\ 11_3$

$201_3 \overline{)\ 12210_3}$

-2010

10200 10

-2010

190 10

-201

212 1

201 1

11

6. $101101_2 \div 11_2$

7. $360E_{12} \times 19T_{12}$

	3	6	0	E	
	0 3	0 6	0 0	0 E	1
6	2 3	6 6	0 0	8 5	9
6	2 6	5 0	0 0	9 2	T
	6	8	2	2	

66822_{12}

8. $307_8 \times 254_8$

$307_8(254_8) = 300_8(254_8) + 7(254_8)$

$= 300_8(200_8) + 300_8(50_8) + 300_8(4_8)$

$+ 7_8(20_8) + 7_8(50_8) + 7_8(4_8)$

$= 60{,}000_8 + 17{,}000_8 + 1400_8$

$+ 160_8 + 430_8 + 34_8$

$= 77000_8 + 1400_8 + 560_8 + 30_8 + 34_8$

$= 100400_8 + 560_8 + 64_8$

$= 101{,}244_8$

100^1400_8

560_8

$+\ 64_8$

$101{,}244_8$

$3 \cdot 2^{100}$

$3^{100} \cdot 2^{100} = (3 \cdot 2)^{100}$

$= 6^{100}$

$2^{50} 2^{19} = 2^{69}$

Exponents: $a^m = \underbrace{a(a)(a)(a)....(a)}_{\text{m times}}$ (repeated multiplication)

Rules of Exponents:

$a^m a^n = a^{m+n}$ ex $5^3 5^6 = 5^9$

$(a^m)^n = a^{mn}$ ex $(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2 = 5^6$

$a^m b^m = (ab)^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ ex $(2 \cdot 3)^5 = 2^5 3^5$

$a^m \div a^n = a^{m-n}$ ex $\frac{4^7}{4^2} = 4^5$

$a^0 = 1$, if $a \neq 0$

ex $2^0 = 1$ or $10^0 = 1$

What is 0^0? undefined

$0^3 = 0$ $3^0 = 1$

$0^2 = 0$ $2^0 = 1$

$0^1 = 0$ $1^0 = 1$

$0^0 = ?$ $0^0 = ?$

$2^3 = 8$ ÷2

$2^2 = 4$ ÷2

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Examples: Simplify.

(a) $(5^7)^2$ $= 5^{14}$

(b) $2^5 2^4$ $= 2^9$

(c) $3^2 4^2$ $= (3 \cdot 4)^2 = 12^2$

(d) $2^7 \div 2^3$ $= 2^4$

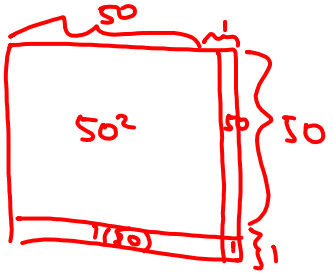
(e) 5^0 $= 1$

HW 3.3A

#16) (a) $51^2 = (50+1)^2 = 50^2 + 1(50) + 1(50) + 1^2$

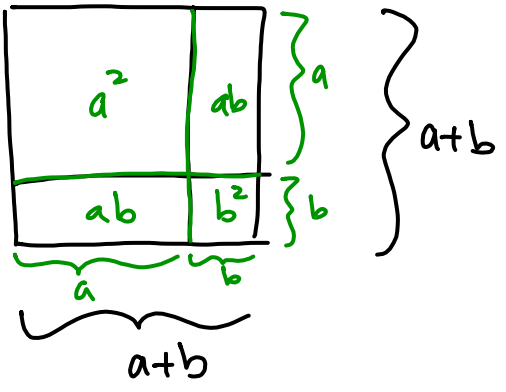
$= 2500 + 100 + 1$

$= 2601$



14b) $(a+b)^2$

$= a^2 + 2ab + b^2$



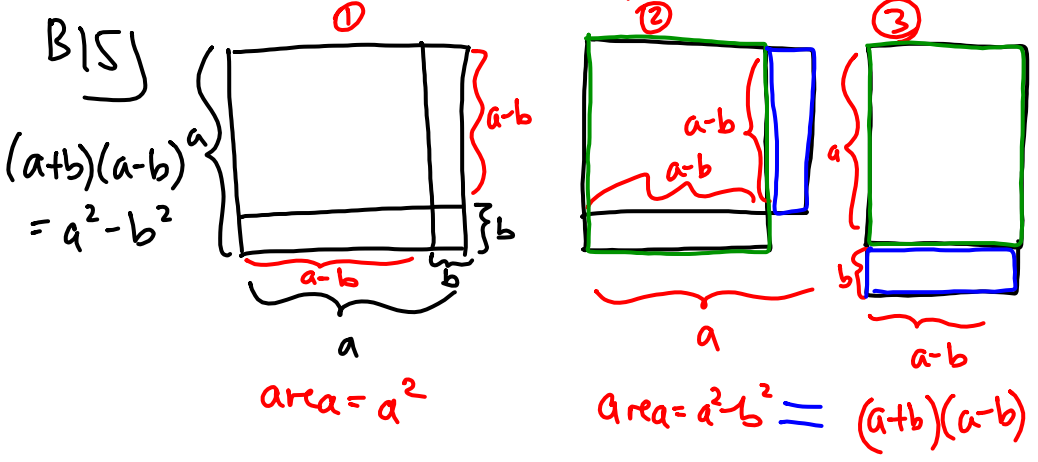
A8) $\frac{6}{\text{shirts}} \cdot \frac{4}{\text{pants}} \cdot \frac{3}{\text{vests}} = 72$

A3) $\{0, 1, 2, 3, 4, 6, 7, 8, 9, 10, \ldots\} = A$ $2+3=5 \notin A$

$\Rightarrow$ not closed under addition

A is closed under multiplication

B15) $(a+b)(a-b) = a^2 - b^2$



3.3 A20) $\frac{5x+5}{5} - 1 = \frac{5(x+1)}{5} - 1$

$= x+1-1 = x$

B2) (b) $\{2k+1 \mid k \in W\} = \{1, 3, 5, 7, 9, 11, \ldots\}$ yes

$m, n \in W$ $(2m+1)(2n+1) = 4mn + 2m + 2n + 1$

$= 2(2mn+m+n)+1$

even

odd

(b) $A = \{2^{k+1} \mid k \in W\} = \{2, 2^2, 2^3, 2^4, \ldots\}$ yes

$2^{n+1} 2^{m+1} = 2^{n+m+2} \in A$, $n, m \in W$

B3c)

⊙	a	b	c
a	a	b	c
b	b	a	c
c	c	c	c

$a \odot a = a$

$a \odot b = b$

$a \odot c = c$

$b \odot a = b$

$b \odot b = a$

(a) yes

(b) yes

(c) identity element = a

(d) test:

$a \odot (b \odot c) \stackrel{?}{=} (a \odot b) \odot c$

$a \odot c \stackrel{?}{=} b \odot c$

$c \stackrel{?}{=} c$ ✓

mc) 4) $\frac{x}{x} = 1$? yes if $x \neq 0$

5) $x \cdot x = x$ yes if $x = 0$ or 1

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ or $x - 1 = 0$

$x = 1$

3.3 MC #13) "1 is identity for division because $a \div 1 = a$ for all whole #s"

false because $a \div 1 \neq 1 \div a$

3.4A #1) (a)

$$\begin{array}{r} {}^{2}4\,\underline{2}\,6 \\ 7\,8\,3 \\ \hline 1\,\underline{2}\,7\,8 \\ {}^{1}\,3\,4\,0\,8 \quad \checkmark \\ \underline{2}\,9\,8\,2 \\ \hline 3\,3\,5\,5\,8 \end{array}$$

$3 \cdot 2 = 6$

$3 \cdot 12 = 36$

3.4A #8)

halves	doubles
27	68
13	136
6	272
3	544
1	1088

$$\begin{array}{r} {}^{2}6\,8 \\ 1\,3\,6 \\ 5\,4\,4 \\ +1\,0\,8\,8 \\ \hline 1\,8\,3\,6 \end{array}$$

A4d) $2^7 \cdot 10^5 \cdot 5^7$

$= (2^7 \cdot 5^7)\,10^5$

$= 10^7\,10^5 = 10^{12}$

17	63
8	126
4	252
2	504
1	1008

$$\begin{array}{r} {}^{1}6\,3 \\ +1\,0\,0\,8 \\ \hline 1\,0\,7\,1 \end{array}$$

63	17
31	34
15	68
7	136
3	272
1	544

1071

3.4A5) (a) $2^{80}+2^{80}$ vs. 2^{100}

$2^{80}+2^{80}=2^1(2^{80})=2^{81}$ $\quad 2^{100}>2^{81}$

(b) $2^{101}, 3(2^{100}), 2^{102}$ $\quad 2^{101}<3(2^{100})<2^{102}$

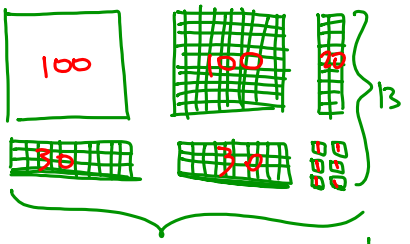
$2^{101}=2(2^{100})<3(2^{100})$

$2^{102}=2^2(2^{100})=4(2^{100})$

$2^{80}+2^{80}\stackrel{?}{\neq}2^{160}$ $\quad 2^3+2^3\neq 2^6$

$8+8\neq 64$

3.4A6)



22·13

$22(13)=(20+2)(10+3)$

$=20(10)+20(3)+2(10)+2(3)$

$=2(100)+3(20)+2(10)+6$

$15(71)$

$=(10+5)(70+1)$

$121=100+2(10)+1$

3.4B#15)

SEND + MORE = MONEY

	S	E	N	D
+	1	0	R	E
1	0	N	E	Y

M=1, O=0

guess S=9

9 2 3 _

+ 1 0 8 2

1 0 3 2 _

guess E=2 ✗

S = 8 or 9

N+R ≥ 12

N = E+1

9 5 6 7

+ 1 0 8 5

1 0 6 5 2

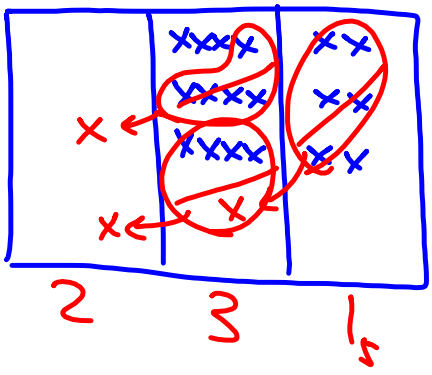
9 3 4 _

+ 1 0 8 3

1 0 4 3 _

guess E=3

3.4B
24) (a) $42_5 \cdot 3_5$



(b) $22_5 \div 4_5 = 3_5$

$22_5 = 2(10)_5 + 2_5$