

3.5 Mental Math and Estimation

Important Note: Estimation is an *approximate* answer to an arithmetic problem.

Some Mental Math Strategies:

1. Use commutativity/associativity/distributivity

ex. $13 + 21 + 45 + 39 + 27$

$$= (13+27) + (21+39) + 45 = 40 + 60 + 45 = 145$$

2. Additive/multiplicative compensation (add zero/multiply by 1)

ex. $98 + 47 = 98+2 + 47-2 = 100 + 45 = 145$

$$64(5) \left(\frac{2}{2}\right) = \frac{64(10)}{2} = 32(10) = 320$$

$$64(5) = 60 \cdot 5 + 4 \cdot 5 = 320$$

3. Special factor

$$\text{ex. } 48(25) = 48(25) \left(\frac{4}{4}\right) = \frac{48}{4}(100) = 1200$$

$$84(5) = 420$$

4. Left-to-right addition

$$\text{ex. } 237 + 436 = 600 + 60 + 13 = 673$$

Some Estimation Strategies:

1. Range estimate

ex. $196(23)$

$$3920 = 196(20) \leq 196(23) \leq 200(23) = 4600$$

2. Compatible number estimation

ex. $89(15) \simeq 90(15) = 1350$

3. Rounding

 ex. $2317 + 4341 \simeq^{(1)} 2320 + 4340$ (to tens digit)
 $= 6660$
 $\simeq^{(2)} 2300 + 4300 = 6600$ (to hundreds digit)

More examples:

$$1. \underbrace{40 + 160} + \underbrace{29 + 31} = 200 + 60 = 260$$

$$2. 75 + 28 = \overset{\textcircled{1}}{(70+20)} + (5+8) \\ = 90 + 13 = 103$$

$$\textcircled{2} 75 + 25 + 3 = 103$$

$$\textcircled{3} 75 + 5 + 28 - 5 = 80 + 23 \\ = 103$$

$$3. 3679 - 474$$

$$\textcircled{4} 78 + 2 + 75 - 2 = 30 + 73 = 103$$

$$\textcircled{1} = 3679 - 474 - 5 + 5$$

$$= 3679 + 5 - 479 = 3200 + 5 = 3205$$

$$\textcircled{2} 3000 - 0 + 600 - 400 \\ + 70 - 70 + 9 - 4$$

$$= 3200 + 5 = 3205$$

$$4. 5075 \div 25 = \overset{\textcircled{1}}{\frac{5000}{25}} + \frac{75}{25} \\ = 200 + 3 = 203$$

$$\textcircled{3} 3600 - 400 + 79 - 74 = 3205$$

$$\textcircled{2} \frac{5075}{25} \left(\frac{4}{4} \right) = \frac{20000 + 300}{100}$$

$$5. 123(3) = 369$$

$$= \frac{20300}{100} = 203$$

$$6. 25 \times 32 \times 4 = (25 \cdot 4) 32 = 3200$$

7. Fliers are being delivered to 3625 houses and there are 42 people who will be doing the distribution. If distributed equally, about how many houses will each person visit?

$$3625 \div 42 \approx 3600 \div 40 = 90$$

Without computing, tell which of the following have the same answer.

(a) $88(44)$ and $44(22)$
 $\neq 22(44)$

(b) $93(15)$ and $31(45)$

$$93(15) = 3(31)(15) = 31(3 \cdot 15) = 31(45)$$

(c) $12(18)$ and $20(17)$
 \neq

Can you "estimate" by calculating the answer exactly and then rounding?

NO

①
$$\begin{array}{r} 25 \\ +37 \\ \hline 62 \end{array}$$
 See I know thirty + twenty = fifty
 seven + five = 12
 fifty + 12 = 62.

Student 1

②
$$\begin{array}{r} 25 \\ +37 \\ \hline 62 \end{array}$$

Student 2

③
$$\begin{array}{r} 25 \\ +37 \\ \hline 62 \end{array}$$

Student 3

④
$$\begin{array}{r} 25 \\ +37 \\ \hline 62 \end{array}$$
 I added the 5 and the 7 together that is 12 so I carried the 1 and put down the 2. $1+2+3=6$. I put down the 6 so it 62.

Student 4

⑤
$$\begin{array}{r} 25 \\ +37 \\ \hline 53 \end{array}$$
 $2+3=5$ and $5+7=12$ so I had 512 I add two to the one and made 53

Student 5

⑥
$$\begin{array}{r} 25 \\ +37 \\ \hline 125 \end{array}$$
 $5+7$ is 12 $2+3$ is 5

Student 6

⑦ What are they thinking?

$$\begin{array}{r} 23 \\ -15 \\ \hline 12 \end{array}$$

⑧
$$\begin{array}{r} 562 \\ -237 \\ \hline 325 \end{array}$$

⑨
$$\begin{array}{r} 562 \\ -287 \\ \hline 185 \end{array}$$

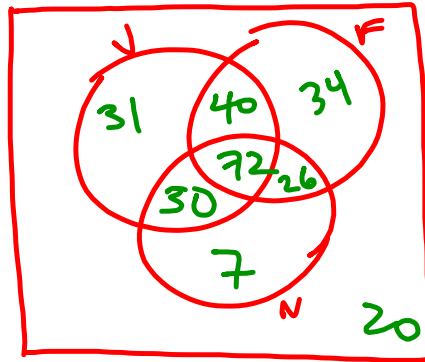
⑩
$$\begin{array}{r} 53 \\ \times 4 \\ \hline 212 \end{array}$$

mid
5)

(b) 31

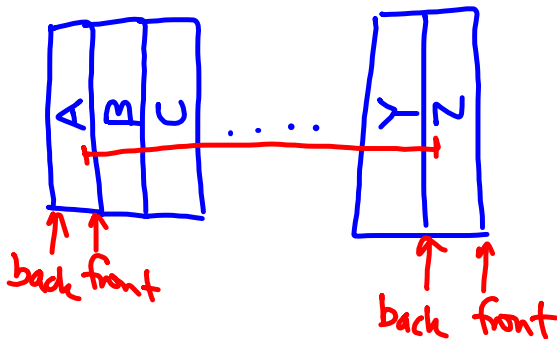
(c) $72 + 26 = 98$

(d) $40 + 30 + 72 + 26 = 168$



- 260 total
- ✓ 173 V
 - ✓ 112 F and V
 - ✓ 125 not N
 - ✓ 72 all three
 - ✓ 88 not F
 - ✓ 51 not F and not N
 - ✓ 20 none

9E)



$$24\left(\frac{3}{2}\right) + 2\left(\frac{1}{8}\right) = 36 + \frac{1}{4} = 36\frac{1}{4} \text{ in}$$

$$7) \quad 49(34) + 66(49) = 49(34) + 49(66) = 49(34 + 66) = 49(100) = 4900$$

$$20(8179.5) = (20 \cdot 5)8179 = 100(8179) = 817,900 = 20 \cdot 8179.5$$

$$2(3.5) = 2(15) = 30 \quad \Bigg| \quad 2(3.5) \neq (2 \cdot 3)(2.5) = 6(10) = 60$$

3.5B

$$b) \quad (a) \quad 74 - 63 = 1 + 10 = 11 \quad \left| \quad \begin{array}{l} 74 - 63 = ? \\ 63 + \underline{\quad} = 74 \end{array} \right.$$

$$(b) \quad 73 - 57 \\ = 10 + 5 + 1 = 16$$

$$\begin{array}{l} 63 + \cancel{7} = 70 \\ 70 + 4 = 74 \\ 7 + 4 = 11 \end{array}$$

MC #4)

$$\textcircled{1} \quad 49 \cdot 51 + 49 \cdot 49 = 4900$$

$$\textcircled{2} \quad 98 \cdot 37 + 2 \cdot 37 = 3700$$

$$\textcircled{3} \quad 99 \cdot 37 + 37 = 3700$$

$$(99+1)37 = 100(37) \uparrow$$

$$\textcircled{1} \quad 49(51+49) \\ = 49(100) \\ = 4900$$

A #11)

$$(a) \quad 2215 + 3023 + 5987 + 975$$

$$\approx 1200 + 200 = 12200$$

$$\text{MC #12) } \underline{\text{ex}} \quad 384(10) = 3840$$

$$384(10) = (3(100) + 8(10) + 4(1))10$$

$$= 3(10^2)10 + 8(10)10 + 4(1)(10)$$

$$= 3(10^3) + 8(10^2) + 4(10) = 3840$$

$$234_5(10_5) = 2340_5$$

$$A2) \quad (a) \quad 2 \cdot 9 \cdot 5 \cdot 6 = (2 \cdot 5)(9 \cdot 6) = 10(54) = 540$$

$$(b) \quad 8 \cdot 25 \cdot 7 \cdot 4 = (25 \cdot 4)(8 \cdot 7) = 5600$$