4.1 & 4.2: Divisibility & Prime/Composite Numbers

Prime number--a natural number with exactly two factors, namely 1 and itself.

Composite number--a natural number with more than two factors.

Question: Is 1 prime or composite?

Divides (wording/notation)

$a \mid b$ (read "a divides b")

other equivalent wording:

- a is a factor of b
- a is a divisor of b
- b is a multiple of a
- b is divisible by a

Ex. $5 \mid 25$ but $3 \nmid 25$ (i.e. 5 divides 25 but 3 does not divide 25)

Tests for Divisibility

2

5

10

3

9

4

8

6

Why does the divisibility test by 9 work? (And, can we extend such a rule to other bases?) For this argument, use a generic five-digit number.

Examples:

(a) Is 7,465,832 a multiple of 4?

(b) Is 8 a factor of 131,888?

(c) Is 497 prime?

(d) Is this true or false? $3 \mid 6n$ for any natural number n

(e) Is this true or false? $0 \mid 0$

(f) True or false: If a and b are both whole numbers, and $5 \nmid a$ and $5 \nmid b$, then $5 \nmid (a + b)$.

(g) Is the number 57,729,364,583 divisible by 2, 3, 5, 6, 8, or 9?

(h) Finish this number so that it is divisible by 9: 123456_ _

(i) I know that 12 pizzas cost $2_0.8_. Can you fill in the missing digits? How much was each pizza?

<u>Fundamental Theorem of Arithmetic</u>

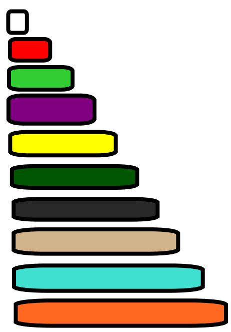
There's only one prime factorization for a composite number.

Prime factorization--

Examples:

(a) Factor 513 completely.

(b) Colored rods are used in many elementary classrooms. The rods vary in length from 1 to 10 cm (in whole number lengths). Various lengths have different colors. A row with all the same color rods is called a one-color train.



- white is 1 unit
- red is 2 units
- light green is 3 units
- purple is 4 units
- yellow is 5 units
- dark green is 6 units
- black is 7 units
- brown is 8 units
- blue is 9 units
- orange is 10 units

(i) What rods can be used to form one-color train for 18?

(ii) What one-color trains are possible for a length of 24?

(iii) If a whole-number length can be represented by an all-red train, an all-green train, and an all-yellow train, what is the least number of factors it must have? What are they?

Ex. Answer these questions about the factors of 97.

(a) If 2 is not a divisor of 97, can any other multiple of 2 be a divisor of 97?

(b) If 3 is not a divisor of 97, can any multiple of 3 be a divisor of 97?

(c) If 5 is not a divisor of 97, what other numbers cannot be divisors of 97?

(d) What numbers must you check to see if 97 has any factors before you decide it's prime?

Ex. Find the prime factorizations for the following numbers.

(a) $36^{10}(49^{20})(6^{15})$

(b) $2(3)(5)(7)(11) + 1$

(c) $2(3^4)(5^{110})(7) + 4(3^4)(5^{110})$

How many divisors does a composite number have?

Let's first try this with a few examples.

(a) 32

(b) 75

(c) $2^3(5^2)(3)$

(d) Is this enough to see a pattern emerging? If so, predict the total number of factors for $2^4(3^5)(5)(7^6)$

Formula: If a composite number has prime factorization of $p_1^n(p_2^m)(p_3^r)$, then it has a total of this many factors: