4.3 Greatest Common Divisor/Factor (GCF) and Least Common Multiple (LCM)

GCF--biggest factor/divisor that two or more numbers have in common

LCM--smallest multiple of two or more numbers

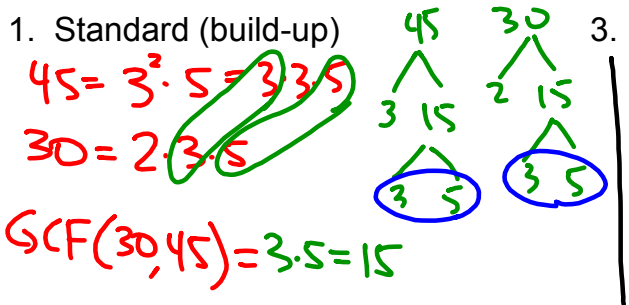
Example: For the numbers 8 and 12, the GCF(8,12) = 4

and LCM(8,12) = 24.

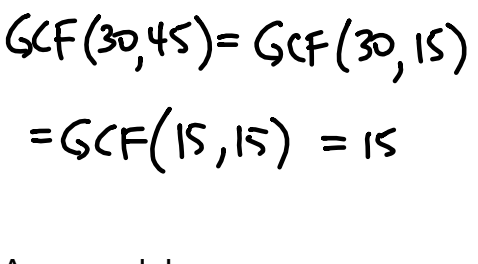
GCF Methods/Algorithms

We'll use the example GCF(45, 30):

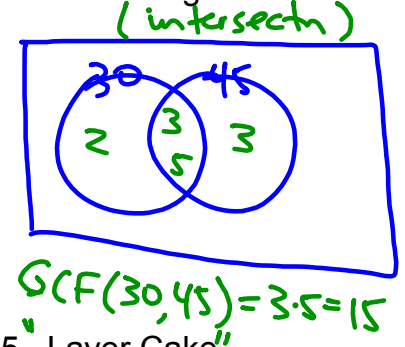
1. Standard (build-up)



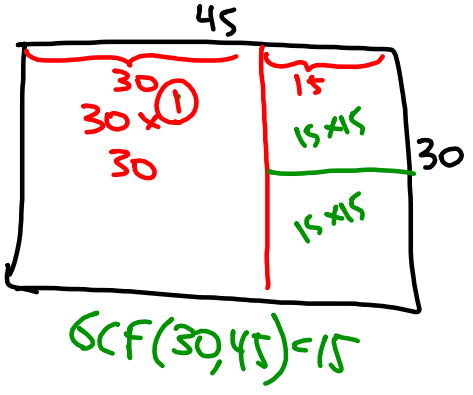
3. Euclidean Algorithm



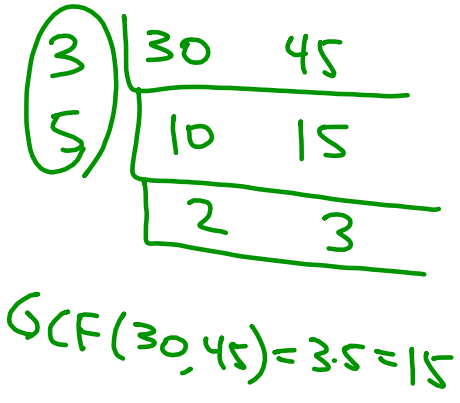
2. Venn Diagram



4. Area model (geometric version of Euclidean)



5. "Layer Cake"



LCM Methods/Algorithms

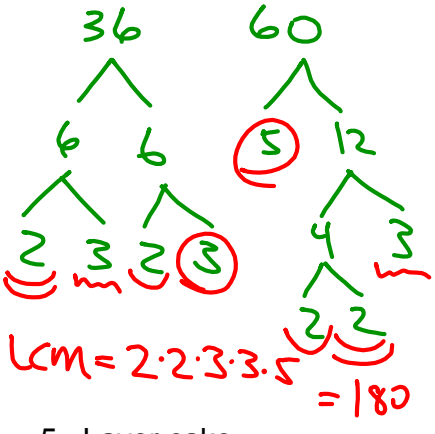
We'll use the example LCM(36, 60):

1. List

multiples of 36:
36, 72, 108, 144, 180, 216, ...

multiples of 60:
60, 120, 180, 240, 300, ...

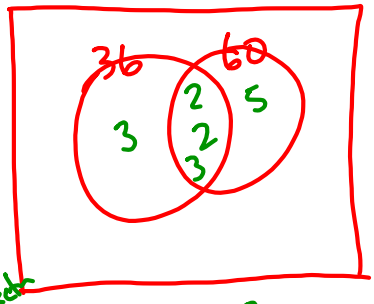
2. Prime Factorization



3. Venn Diagram

$36 = 2^2 \cdot 3^1$

$60 = 5 \cdot 2^2 \cdot 3^1$



intersect GCF = 12 = 2·2·3

union LCM = 3·2·2·3·5 = 180

36·60 = 180(12) ✓

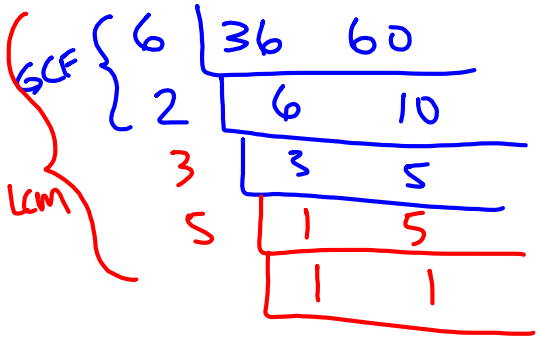
4. Area model, along with fact

GCF(a,b) * LCM(a,b) = ab

know GCF(a,b)

$LCM(a,b) = \frac{ab}{GCF(a,b)}$

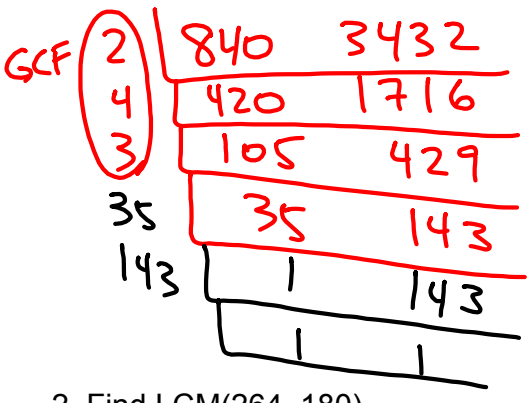
5. Layer cake



LCM = 6·2·3·5 = 180

Examples:

1. Find GCF(840,3432)



$LCM = 2 \cdot 4 \cdot 3 \cdot 35 \cdot 143$

$GCF = 24$

$GCF(840, 3432)$

$= GCF(840, 912)$

$= GCF(840, 72)$

$= GCF(72, 120)$

$= GCF(72, 48)$

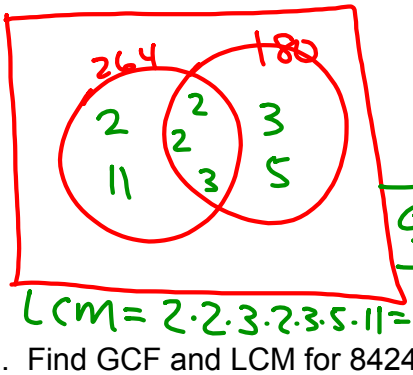
$= GCF(48, 24) = GCF(24, 24)$

$= 24$

$3432 - 2520 = 912$

$840 - 720 = 120$

2. Find LCM(264, 180)



$GCF = 2 \cdot 2 \cdot 3$

$LCM = 2 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \cdot 5 \cdot 11 = 2^3 \cdot 3^2 \cdot 5 \cdot 11$

3. Find GCF and LCM for 8424 and 2520

$GCF(8424, 2520) = GCF(2520, 864)$

$= GCF(864, 792) = GCF(792, 72)$

$= GCF(72, 72) = 72$

4. Find GCF and LCM for 4^{100}, 50^{20} and 20^{35}

$(2^2)^{100}$ $(2 \cdot 25)^{20}$ $(2 \cdot 5^2)^{20}$

2^{200} $2^{20} \cdot 5^{40}$ $2^{70} \cdot 5^{35}$

$GCF(4^{100}, 50^{20}, 20^{35}) = 2^{20}$

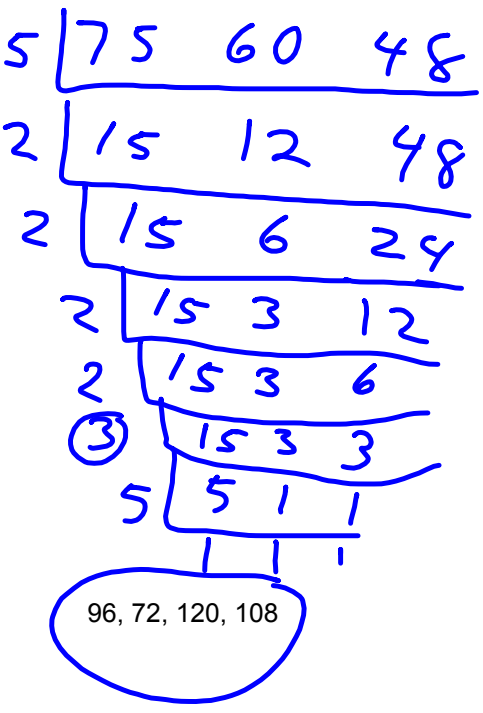
$LCM(4^{100}, 50^{20}, 20^{35}) = 2^{200} \, 5^{40}$

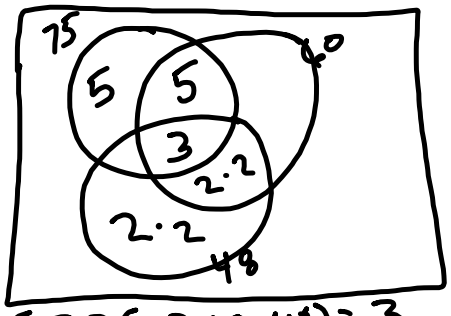
Which methods work for three numbers?

$75: 3 \cdot 5^2$

⋮

Determine the GCF and LCM of 75, 60 and 48





$GCF(75, 60, 48) = 3$

$LCM(75, 60, 48) = 2^4 \cdot 3 \cdot 5^2$

$75 = 3 \cdot 5^2$

$60 = 2^2 \cdot 5 \cdot 3$

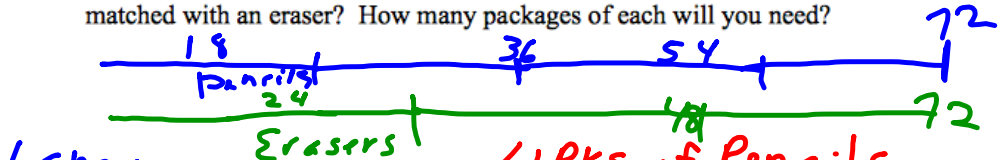
$48 = 2^4 \cdot 3$

$2^4 \cdot 3 \cdot 5^2$

(From your homework:)

GCF and LCM Problems

1. Pencils come in packages of 18. Erasers that fit on top of these pencils come in packages of 24. What is the smallest number of pencils and erasers that you can buy so each pencil can be matched with an eraser? How many packages of each will you need?



LCM(18,24) = 72

4 Pks of Pencils
3 Pks of erasers.

2. Ko has a bag with 45 red candies and another with 75 green candies. She wants to make goody bags so that each goody bag contains the same number of red candies and the same number of green candies and so that she uses up all of the candies. What is the largest number of goody bags she can make this way? How many of each color candy will be in each goody bag?

GCF(45,75) = 15

3R
5G in each of 15 bags.

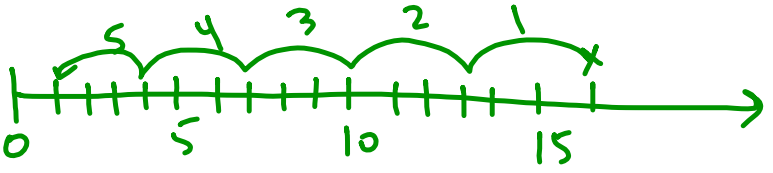
3. Same has lots of 8-inch sticks he is placing end to end to make a line of sticks. Becky has 12-inch sticks that she is placing end to end as well. If they want their line of sticks to be the same length, how long could they be? What is the shortest such length?

LCM(8,12) = 24

24, 48, ...

Quiz 7

2) (b) $16 \div 3 = ?$



4) (c) $8706 - 87$

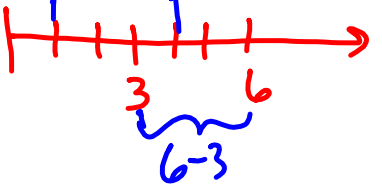
$-6 \quad +6$

$8706 - 87 \neq 8706 - 6 - (87 + 6)$

$= 8706 - 6 - 87 - 6$

$8706 - 6 - (87 - 6)$

ex $6 - 3 = (6 - 2) - (3 - 2)$



0^0 is undefined

0 to any power is supposed to be 0, but anything to the power of 0 is supposed to be 1. Since we have 2 conflicting results, 0^0 is ill-defined.

Quiz 8

1) (a) No, ex 12

(b) Yes. because 8 and 3 have no common factors.

- a # is divisible by 30 iff it's divisible by both 5 an 6

true

- a # is divisible by 30 iff it's divisible by both 2 and 15

true

- a # is divisible by 18 iff it's divisible by both 6 and 3

No, ex 12

2) (b) $15 \mid (2^4 \cdot 3^5 \cdot 5)$

$= 15 \mid (3 \cdot 5)(2^4 \cdot 3^4)$

3) Is 1 a prime #?

NO

4) (b) $2^2 \cdot 3^5 \cdot 7^{55} + 2^4 \cdot 3^4 \cdot 7^{55}$

$= 2^2 \cdot 3^4 \cdot 7^{55}[3 + 2^2]$

$= 2^2 \cdot 3^4 \cdot 7^{55}[7] = 2^2 \cdot 3^4 \cdot 7^{56}$

7) GCF = 1

6) GCF(1575, 1960)

$= GCF(1575, 385)$

$= GCF(385, 1190) = GCF(385, 420)$

$= GCF(385, 35) = GCF(35, 35) = 35$

$1960 - 1575 = 385$

$1190 - 770 = 420$

$385 - 350 = 35$

$1575 \overline{)1960}$ (1) -1575, 385

$385 \overline{)1575}$ (4) -1540, 35

$35 \overline{)385}$ (11) -385, 0