5.1 Addition and Subtraction of Integers

The set of integers = **Z** = $\{\ldots,-3,-2,1,0,1,2,3,\ldots\}$

$\mathbb{Z}$

Properties for Integers with Addition

1. Closure "$\mathbb{Z}$ is closed under addition"
   any 2 integers add to give another integer
2. Commutativity
   $a+b=b+a$
3. Associativity
   $(a+b)+c=a+(b+c)$
4. Additive Identity
   $0+a=a+0=a \quad a\in\mathbb{Z}$
5. Additive Inverse
   $a+^{-}a=0=-a+a$ ex $a=3$
   $3+^{-}3=0$

(closed under subtraction)

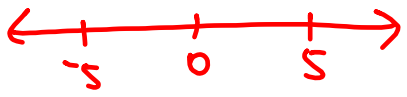
- sign...its many guises and names

1. subtraction sign

ex 7-4 "7 minus 4"

2. negative sign

ex -5



3. opposite

ex $-(-4)=4$ $-4-3=-7$

$-(-(-(5)-3$

"opp of opp of opp of neg 5 minus 3"

1. How would you properly read these statements? And can you explain why these are true?

(a) -(-x) = x

ex $x=-2$

"opp of opp of x is x" $-(-x)=-(-(-2))=-2$

(b) -a + -b = -(a + b)

ex $a=3, b=-4$ $-a+-b=-3+4$

$=-(3+-4)$

" opp of a + opp of b is opp of sum a + b"

(c) a - b = a + (-b)

"a minus b is a plus opp of b"

2. Is -x always negative? no

ex $x=4$, $-x=-4$ ex $x=-4$, $-x=4$

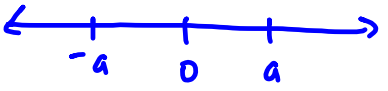
ex $x=0$, $-x=0$

Absolute Value Definition

$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

ex $|-3| = -(-3) = 3$

$|-3| = 3$



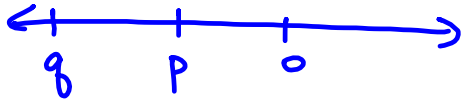
$|a| = |-a| = a$, if $a \geq 0$

Geometrically, the absolute value of a number represents how far away it is from the origin on the real number line.

1. Explain whether the sum of any two negative numbers is also negative.

$p, q < 0$

$p + q < 0$



p + q gives some value further to the left of 0

2. Explain whether the sum of a positive integer with a negative integer is positive or negative and why?

it depends ex $-4 + 5 = 1$ ex $-3 + 3 = 0$

ex $-5 + 4 = -1$

3. Simplify.

(a) $|x| + x$ if $x < 0$

$= -x + x = 0$

ex $x = -1$

$|x| + x = |-1| + -1$

$= 1 + -1 = 0$

(b) $-|x| + x$ if $x < 0$

$= -(-x) + x = x + x = 2x$

ex $x = -9$

$-|x| + x = -|-9| + -9$

$= -(9) + -9 = -9 + -9$

$= 2(-9)$

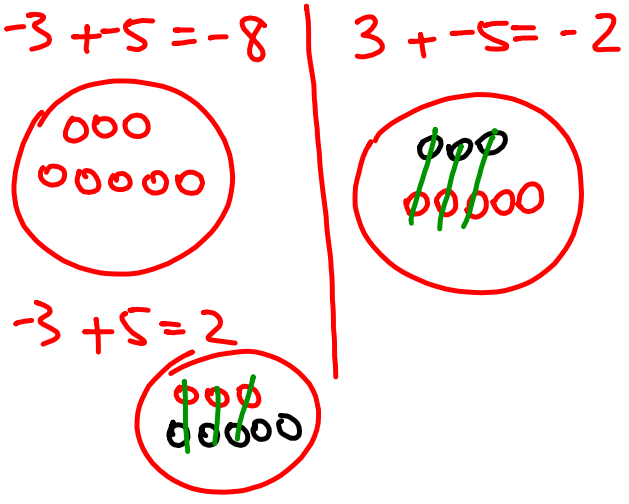
(c) $-|x| + x$ if $x > 0$

$= -(x) + x = 0$

Addition of Integers--various models/algorithms

O = negative one
O = positive one

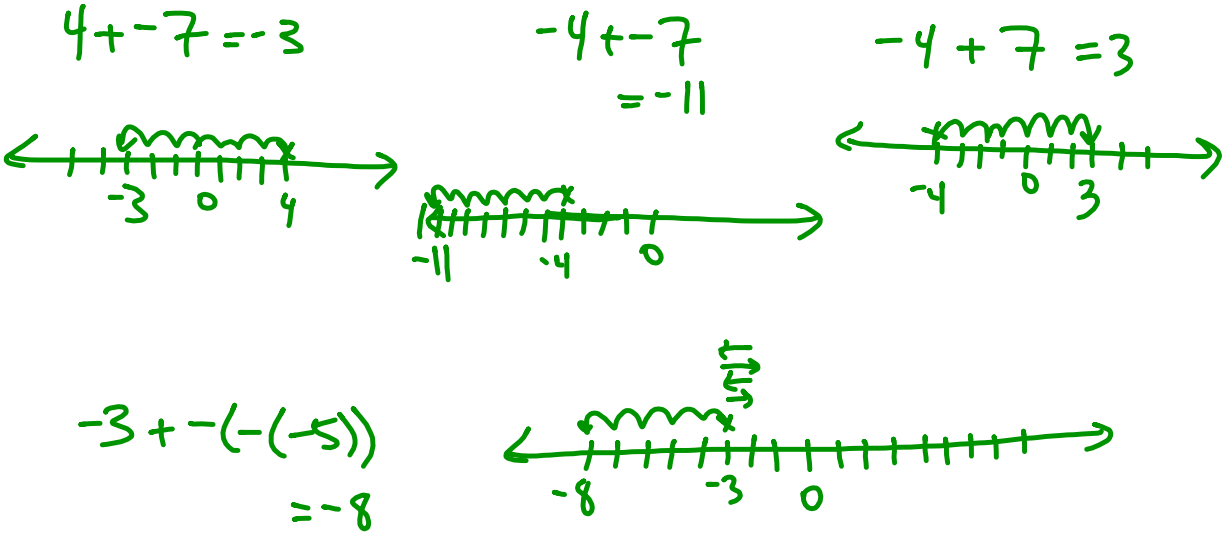
1. Set Model



3. Pattern 2 + -4

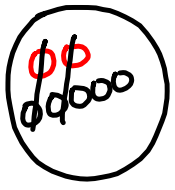
2 + 2 = 4
2 + 1 = 3
2 + 0 = 2
2 + -1 = 1
2 + -2 = 0
2 + -3 = -1
2 + -4 = -2

2. Measurement (number line)



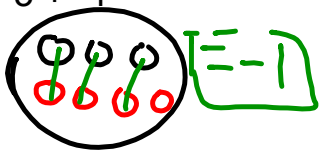
Examples: O=positive
O=negative

set
1. -2 + 5

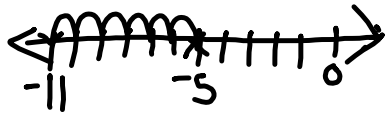


-2+5=3

Set
2. 3 + -4



line
3. -5 + -6 = -11



4. Make up a story problem that would produce this addition computation.

23 + -15 + -8

(a) (money) I have $23. I bought a t-shirt for $15, and another t-shirt for $8. How much $ do I have now?

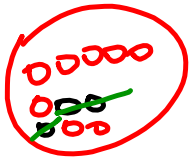
(b) (sea level) Start at 23 feet above sea level. Add a jump down of 15 ft and another jump down by 8 ft.

Subtraction of Integers--various models/algorithms

1. Set Model

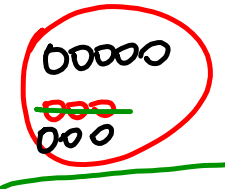
$-5-(-3)=-2$

$-5-3=-8$



3. Pattern

$5-(-3)=8$



$-3-(-1)=-2$

$-3-0=-3$

$-3-1=-4$

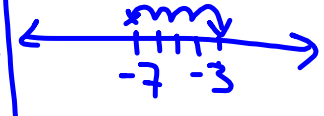
$-3-2=-5$

$-3-3=-6$

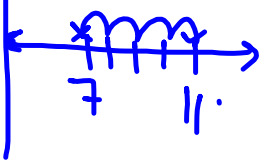
2. Measurement (number line)

$-7-4=-11$

$-7-(-4)=-3$



$7-(-4)=11$



4. Adding the opposite

ex) $-8-(-4)$

$=-8+4=-4$

ex) $-5-1=-5+(-1)$

$=-6$

5. Missing Addend

$-3-5=?$

$\Leftrightarrow -3=5+?$

$?=-8$

$4-9=?$

$\Leftrightarrow 4=9+?$

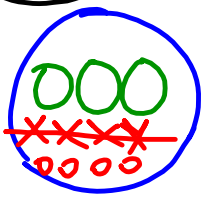
$?=-5$

Examples: O positive
O negative

1. -2 - 5 = -7

set

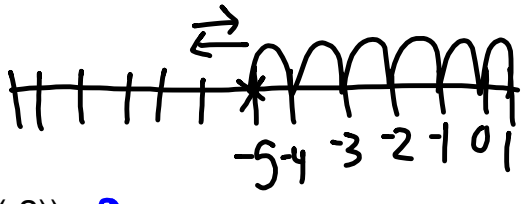
2. 3 - -4 = 7



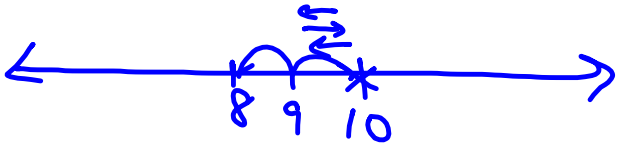
O = positive one
X = negative one

3. -5 - -6 = 1

#line



4. 10-(-(-2)) = 8



5. Make up a story problem that would produce this computation.

23 - (-17) = 40

I have $23. My friend remembers he owes me $17 & pays it back to me.

5.1A #7) (a) $3-(-2)=?$ $\Leftrightarrow$ $3=-2+?$

A#17) W = whole #s, I = integers, I^+ = pos. integers

I^- = neg. integers

(a) $W \cup I = I$

(b) $W \cap I = W$

(c) $I^+ \cup I^- = I - \{0\}$

(d) $I^+ \cap I^- = \emptyset$

B 22) $y = |x-6|$

$$y = \begin{cases} x-6 & \text{if } x-6 \geq 0 \\ -(x-6) & \text{if } x-6 < 0 \end{cases}$$

$$= \begin{cases} x-6 & \text{if } x \geq 6 \\ -x+6 & \text{if } x < 6 \end{cases}$$

defn $y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

5.1 BH) (a) $W - I^+ = \{0\}$ (c) $I \cap I = I$

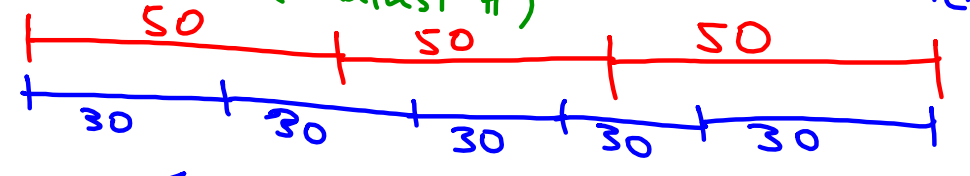
(b) $W - I^- = W$ (d) $I - W = I^-$

GCF/LCM Wksht:

#6) red lights 50/string; blue lights 30 per string

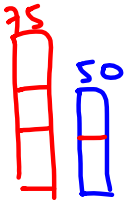
(a) how often do I need hangers? (smallest #)

LCM(30, 50) = 150



GCF(30, 50) = 10

#7)



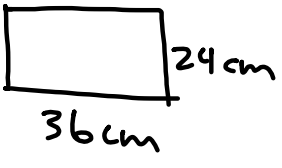
(a) tall as possible both have same ht

GCF(75, 50) = 25

(b) least # per box (single color)

LCM(75, 50) = 150

5)



LCM(36, 24) = 72