5.2 Multiplication and Division of Integers

Properties for Integers with Multiplication

1. Closure the product of any 2 integers is also an integer

2. Commutativity $ab=ba$
   (order doesn't matter for multiplication)

3. Associativity
   $a(bc)=(ab)c$ (grouping doesn't matter)

4. Multiplicative Identity
   $1 \in \mathbb{Z}$

5. Distributivity
   $a(b \pm c)=ab \pm ac$

6. Zero Multiplication Property
   If $ab=0$, then $a=0$ or $b=0$.

1. How would you properly read these statements? And can you explain why th

(a) (-1)a = -a = a(-1) true

(b) -a(b) = -(ab) = a(-b) true

(c) (-a)(-b) = ab = -(-(ab)) true

$-(-(-(-(-(ab)))))=-ab$

2. A little more about absolute value. Fill in the blank with <, =, or >.

(a) |a| + |b| ≥ |a + b|

ex $a=5, b=-2$

$5+2 \overset{?}{\geq} |5+-2|$ ✓

ex $a=-3, b=-4$

$|-3|+|-4| = |-3+-4|$

$3+4=7$ ✓

(b) |a| (|b|) = |a(b)|

(c) |a| - |b| ≤ |a - b|

ex $a=5, b=2$

$5-2 = |5-2|$

ex $a=-5, b=2$

$5-2 < |-5-2|$

7

ex $a=5, b=-2$

$5-2 < |5-(-2)|$

(d) |a| ÷ |b| = |a ÷ b|

ex $a=-5, b=-2$

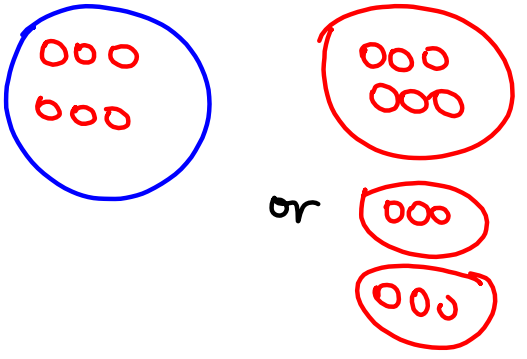
$5-2 = |-5-(-2)|$

Multiplication of Integers--various models/algorithms

1. Set Model

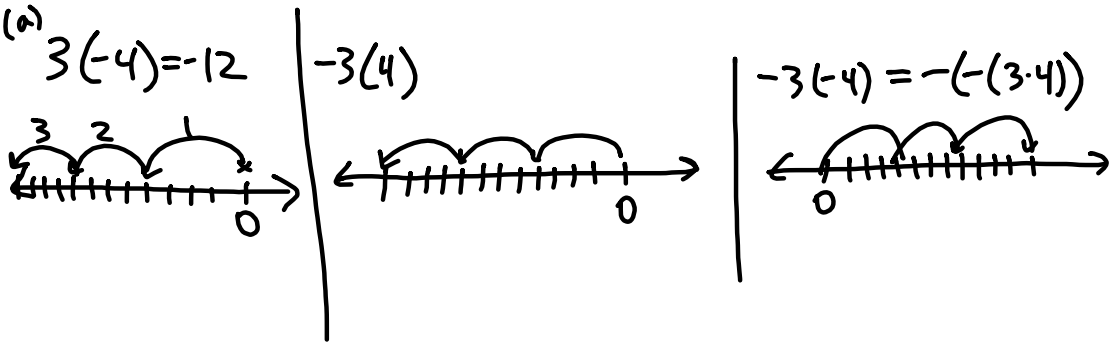
0 = negative one
0 = positive one

$2(-3)=-6$ $-2(-3)=6$



2. Measurement (number line)

(a) $3(-4)=-12$ $-3(4)$ $-3(-4)=-(-(3\cdot4))$



3. Pattern

$4(-2)=?$

$4(2)=8$
$4(1)=4$
$4(0)=0$
$4(-1)=-4$
$4(-2)=-8$

$4(-2)$
$=-4\cdot2$
$=-(4\cdot2)$
$=-8$

4. Repeated Addition

$5(-3)=-3+-3+-3+-3+-3$
$=-15$

$0\cdot0=0$
$0\cdot0=0$
$0\cdot0=0$

5. Area Model

$2(-3)$
$=-6$

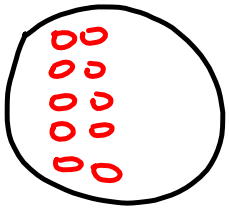
	0	0	0
0	0	0	0
0	0	0	0

$-2(-3)$
$=6$

	0	0	0
0	0	0	0
0	0	0	0

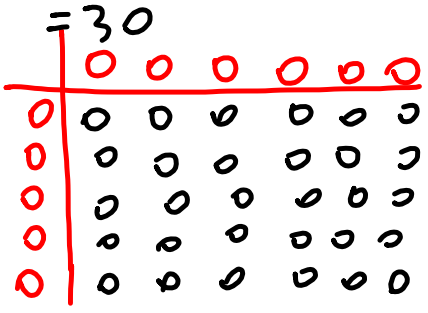
Examples:

1. -2(5) $= 5(-2) = -10$



2. 3(-4) $= -12$

3. -5(-6) $= 30$



4. Make up a story problem that would produce this computation.

   8(-9)

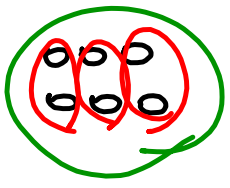
Eight people each owe me $9.

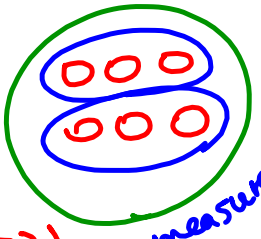
Division of Integers--various models/algorithms

a ÷ b = ? is equivalent to a = b(?) (assuming b is not zero) (missing factor)

1. Set Model

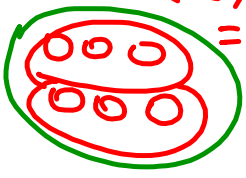
$6 \div (-3) = -2$ $-6 \div 3 = -2$





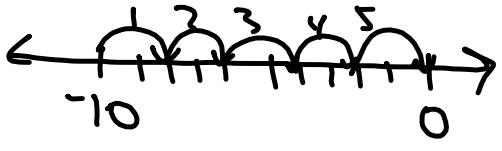
measurement

Partitive $-6 \div (-3) = 2$

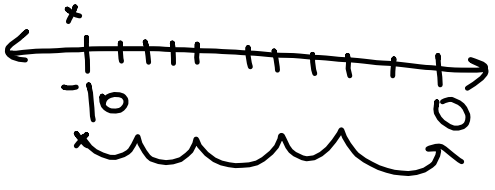


2. Measurement (number line)

$-10 \div 2 = -5$



$-10 \div -2 = 5$



3. Pattern

$15 \div (-3) = ?$

$15 \div 3 = 5$

$15 \div (-1) = -15$

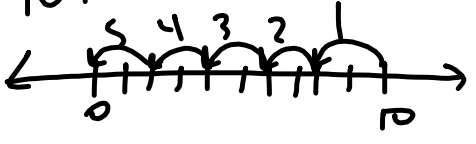
$15 \div (-3) = -5$

4. Missing Factor

$6 \div (-3) = ?$

$\Leftrightarrow 6 = -3 \cdot ?$

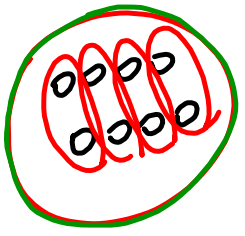
$10 \div 2$



Examples:

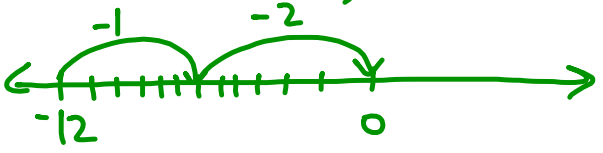
O = neg
O = pos

1. $8 \div (-2)$ (set)



$= -4$

2. $-12 \div 6$ (# line) $= -2$

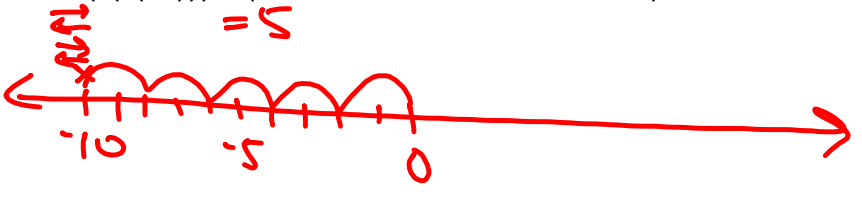


3. $-15 \div (-3) = ?$

$\Leftrightarrow -15 = -3 \cdot ?$

$? = 5$

4. $-10 \div (-(-(-2)))$ (show on the number line)



$= 5$

5. Make up a story problem that would produce this computation.

$-25 \div 5$

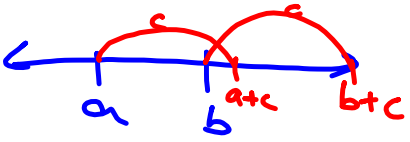
I owe a total of $25 to be split equally among 5 friends. How much do I owe each friend?

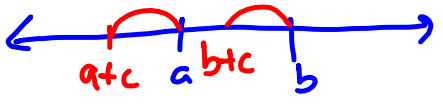
Ordering Integers

1. If a < b and b < c, then a $\leq$ c.

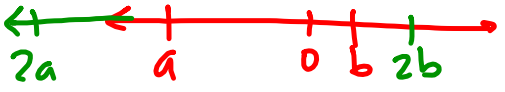
(transitivity)

2. If a < b, then a + c $\leq$ b + c.

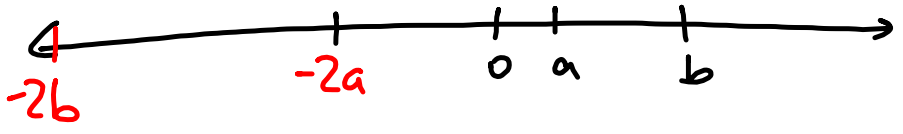




3. If a < b, then ap $\leq$ bp, assuming p > 0.



4. If a < b, then an $\geq$ bn, assuming n < 0.



$-3 < 5$

$(-1)(-3) \;?\; 5(-1)$

$3 > -5$

5.2A #15) $x \neq 0, x \in \mathbb{Z}$

(a) $-x^2$ neg.

(b) x^2 pos.

(c) $(-x)^2$ pos.

(d) $-x^3$

(e) $(-x)^3$ $=$ } sometimes pos / sometimes neg.

MC #3) $(-1)a = -a \Rightarrow (-1)(ab) = -(ab)$ "opposite identity property used"

and $(-1)(ab) = (-1 \cdot a)b$ associativity

$= -(a)b$

$\Rightarrow (-a)b = -(ab)$ ✓

MC 6) $$\frac{-2(-3x+2)-14}{6} = \frac{6x-4-14}{6}$$

$$= \frac{6x-18}{6} = x-3 \Rightarrow$$ add 3 to get back to x

B24) (a) $(a-1)^2 = (a-1)(a-1) = a^2 - a - a + 1$

$= a^2 - 2a + 1$

$(a-1)(a-1) = a(a-1) - 1(a-1)$

(b) $199^2 = (200-1)^2 = 200^2 - 2(200) + 1 = 40000 - 400 + 1$

$=$