6.1 Rational Numbers

set of rational numbers = **Q** =

Vocabulary--

numerator

denominator

proper fraction

improper fraction

We use fractions in two ways:

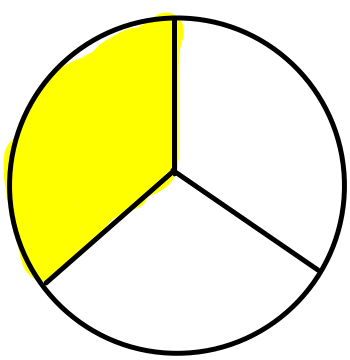
1. part-to-whole

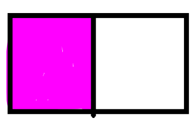
We need to consider: (a) the whole, (b) the number of equal-sized parts that the whole has been divided into, and (c) the number of parts we have.

2. relative amount

Draw a Venn Diagram to display the relationship between the natural numbers, whole numbers, integers and rational numbers.

Max claims that $\frac{1}{3} > \frac{1}{2}$ because in the below figure, the shaded portion for $\frac{1}{3}$ is larger than the shaded portion depicting $\frac{1}{2}$. Is he correct? It not, how would you help him?





Equivalent fractions==> fractions that represent the same relative amount

$$\frac{a}{b}=\frac{an}{bn} \text{ for any nonzero n}$$

How to decide if fractions are equal:

$$\frac{a}{b}=\frac{c}{d} \text{ iff } ad=bc \text{ (assuming } b\neq 0 \text{ and } d\neq 0)$$

Other ideas?

Ex 1. Are these true or false statements? Why?

(a) $\frac{16}{56}=\frac{2}{7}$

(b) $\frac{2}{6}=\frac{1}{4}$

Ex 2. Create three other equivalent fractions for $\frac{4}{9}$.

Ordering fractions:

1. $\frac{a}{c} < \frac{b}{c}$ iff $a < b$

2. $\frac{a}{b} > \frac{c}{d}$ iff $ad > bc$ (assuming b, d > 0)

3. If $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$ (assuming that b, d > 0).

Ex 3. Order these rational numbers from least to greatest and plot them on a number line.

(a) $\frac{4}{7}, \frac{9}{10}, \frac{8}{9}, \frac{1}{4}, \frac{2}{5}, \frac{5}{6}$

(b) $\frac{3}{4}, \frac{9}{16}, \frac{5}{8}, \frac{2}{3}, -\frac{3}{8}, -\frac{6}{11}, -\frac{4}{9}$

Ex 4. (a) Is this true or false and why? $\frac{7}{8} < \frac{10}{11}$

(b) Tell whether each of these fractions is closer to 0, one-half or 1.

$\frac{3}{8}, \frac{2}{7}, \frac{1}{3}, \frac{21}{50}, \frac{4}{5}, \frac{7}{11}, \frac{31}{181}, \frac{3}{4}$

(c) Fill in the blank with < , > or =. $\frac{7}{8}$ _____ $\frac{5}{9}$

Simplifying Fractions

A rational number, a/b, is in simplest form iff the GCF(a,b) = 1, assuming b is nonzero.

Ex 5. Simplify these fractions.

(a) $\frac{12}{18}$

(b) $\frac{42}{52}$

(c) $\frac{294}{63}$

(d) $\frac{2^2 3^4 5^3}{2^3 3 \cdot 5^2}$

(e) $\frac{14\mathrm{a} b^2}{20\mathrm{a}^5 b^3}$

(f) $\frac{8+x^2}{2\mathrm{x}}$

Explain why there are infinitely many rational numbers between any two rational numbers.