6.3 Multiplying/Dividing Fractions (Rational Numbers)

Properties for Rational Numbers with Multiplication

1. Closure

2. Commutativity

3. Associativity

4. Multiplicative Identity $\quad a \cdot 1 = a = 1 \cdot a \qquad\qquad 1 \in \mathbb{Q}$

★ 5. Multiplicative Inverse $\quad a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1, \; a \neq 0$

if $a \in \mathbb{Q}$ (but not 0), then $\frac{1}{a} \in \mathbb{Q}$.

ex $\quad \frac{-3}{5} \cdot \frac{-5}{3} = 1$

To multiply fractions: $\quad \dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}\qquad$ (simplify first, before multiplying)
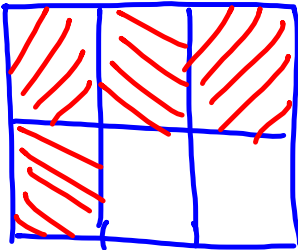
ex $\quad \dfrac{\cancel{5}^{\,1}}{\cancel{8}_{\,2}} \cdot \dfrac{\cancel{4}^{\,1}}{\cancel{9}_{\,3}} \cdot \dfrac{\cancel{3}^{\,1}}{\cancel{25}_{\,5}} = \dfrac{1 \cdot 1 \cdot 1}{2 \cdot 3 \cdot 5} = \dfrac{1}{30}$

Multiplication Models

1. Repeated Addition (only works if one of the factors is a whole #)
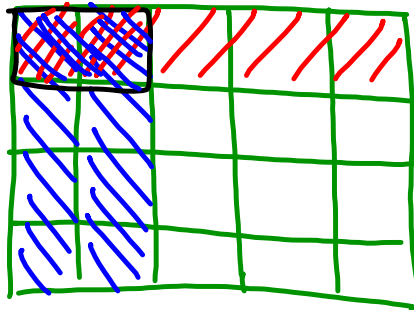
$4 \cdot \frac{1}{6}$

"4 groups of $\frac{1}{6}$"

$$4 \cdot \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

2. Area model (handy if you have 2 proper fractions to multiply)

$\frac{1}{4} \times \frac{2}{5}$

$$\frac{1}{4} \times \frac{2}{5} = \frac{2}{20}$$

Division with Fractions

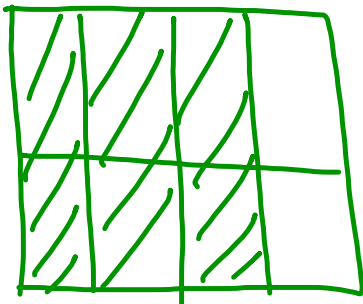To divide fractions: $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{ad}{bc}$ but why?

Use $\dfrac{3}{4} \div \dfrac{1}{8}$ as a starting argument.

① $\dfrac{3}{4} \div \dfrac{1}{8} = ?$ $\Longleftrightarrow$ $\dfrac{3}{4} = ? \cdot \dfrac{1}{8}$ $? = 6$

$$\dfrac{3}{4} = \boxed{\dfrac{3 \cdot 2}{1}} \cdot \dfrac{1}{8}$$

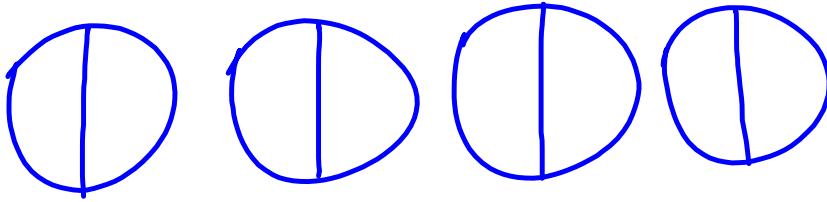② $\dfrac{3}{4} \div \dfrac{1}{8} = \dfrac{\frac{3}{4}}{\frac{1}{8}} \left(\dfrac{\frac{8}{1}}{\frac{8}{1}} \right) = \dfrac{\frac{3}{4} \cdot \frac{8}{1}}{1} = \dfrac{3}{4} \cdot \dfrac{8}{1}$

③ $\dfrac{3}{4} \div \dfrac{1}{8} = \dfrac{3}{4} \left(\dfrac{2}{2} \right) \div \dfrac{1}{8} = \dfrac{6}{8} \div \dfrac{1}{8} = 6 \div 1 = 6$

"out of $\dfrac{6}{8}$ (shaded),
how many $\dfrac{1}{8}$'s are there?"
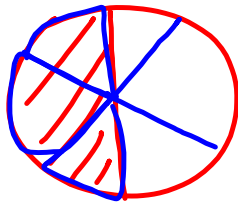
$4 \div \dfrac{1}{2}$ means "how many groups of one-half are there in 4?"

$4 \div \dfrac{1}{2} = 8$

$\dfrac{1}{2} \div \dfrac{1}{6}$ means:

$= 3$

"A recipe calls for $\frac{1}{2}$ cup sugar. I only have $\frac{1}{6}$-cup measuring cup. How many of those $\frac{1}{6}$-cups must I fill to make my recipe?"

Examples:

1. $\dfrac{50 \cdot 39 \cdot 5}{15 \cdot 55 \cdot 2} = \dfrac{5 \cdot 13 \cdot 1}{1 \cdot 11 \cdot 1}$

$= \dfrac{65}{11}$

2. $\dfrac{2 \cdot 32}{4 \cdot 18} = \dfrac{8}{6} = 1\dfrac{2}{6} = 1\dfrac{1}{3}$

or $\dfrac{4}{3}$

3. $\dfrac{23 \cdot 22}{4 \cdot 3} = \dfrac{11}{4} \cdot \dfrac{8}{3}$

$= \dfrac{22}{3}$ or $7\dfrac{1}{3}$

4. $\dfrac{4}{11} \div \dfrac{7}{22} = \dfrac{4}{11} \cdot \dfrac{22}{7} = \dfrac{8}{7}$ or $1\dfrac{1}{7}$

$\div \dfrac{11}{4} \cdot \dfrac{7}{22}$

5. $\dfrac{6}{13} \div \dfrac{2}{39} = \dfrac{6}{13}\left(\dfrac{3}{3}\right) \div \dfrac{2}{39} = \dfrac{18}{39} \div \dfrac{2}{39} = 18 \div 2 = 9$

6. $1\dfrac{3}{4} \div \dfrac{9}{8} = \left(1 + \dfrac{3}{4}\right) \div \dfrac{9}{8}$

$= \dfrac{7}{4} \div \dfrac{9}{8} = \dfrac{7}{4} \cdot \dfrac{8}{9} = \dfrac{14}{9}$

or $1\dfrac{5}{9}$

7. $\dfrac{6}{25} \div \dfrac{3}{5}$

$= \dfrac{6}{25} \cdot \dfrac{5}{3}$

$= \dfrac{2}{5}$

Students often confuse dividing by 2 with dividing by 1/2.
Write two story problems that show the difference.

dividing by 2

① Class of 28 students. Divide into 2 equal-sized teams. How many on each team?

$28 \div 2$

dividing by ½

② class of 28 students. Divide each student in half. How many half-students are there?

6.3 HW

A4) (a) $4\frac{1}{2} \cdot 2\frac{1}{3}$

$$= \left(4 + \frac{1}{2}\right)\left(2 + \frac{1}{3}\right)$$

$$\left(\frac{3}{2}\right)\frac{4}{3} + \frac{1}{6} = \frac{8+1}{6}$$
$$= \frac{9}{6} = \frac{3}{2}$$

$$= 4(2) + 4\left(\frac{1}{3}\right) + \frac{1}{2}(2) + \frac{1}{2}\left(\frac{1}{3}\right) = 8 + \frac{4}{3} + 1 + \frac{1}{6}$$
$$= 9 + \frac{3}{2} = \boxed{10\frac{1}{2}}$$

A6d) $\dfrac{2x}{3} - \dfrac{1}{4} = \dfrac{x}{6} + \dfrac{1}{2}$

$$12\left(\frac{2x}{3} - \frac{1}{4}\right) = \left(\frac{x}{6} + \frac{1}{2}\right)12$$

$$\overset{4}{12}\left(\frac{2x}{3}\right) - \overset{3}{12}\left(\frac{1}{4}\right) = \frac{x}{6}\overset{2}{(12)} + \frac{1}{2}\overset{6}{(12)}$$

$$8x - 3 = 2x + 6$$
$$6x = 9$$
$$x = 9/6 = \boxed{3/2}$$

A15) $x =$ original amt of $ in acct.

$$\underbrace{50}_{\text{radio}} + \underbrace{\frac{3}{5}(x-50)}_{\text{gifts}} + \underbrace{\frac{1}{2}(\text{"what's left"})}_{\text{checking}} + \underbrace{35}_{\text{charity}} = x$$

$$\text{"what's left"} = x - \underbrace{\frac{3}{5}(x-50) - 50}$$

$$50 + \frac{3}{5}x - 30 + \frac{1}{2}\left(x - \frac{3}{5}x + 30 - 50\right) + 35 = x$$

$$\frac{1}{2}\left(x - \frac{3}{5}x + 30 - 50\right) = 35$$

$$\frac{2}{5}x + -20 = 70$$

$$\frac{5}{2} \cdot \frac{2}{5}x = 90 \cdot \frac{5}{2}$$

$$x = 45(5) = 225$$

$$x = \boxed{\$225}$$

B11) $\quad 3p - \frac{7}{18} = 2p + \frac{5}{12}$

$\quad\quad\quad p = \frac{5}{12} + \frac{7}{18} \quad\quad\quad\quad LCD = 36$

$\quad\quad\quad p = \frac{5}{12}\left(\frac{3}{3}\right) + \frac{7}{18}\left(\frac{2}{2}\right) = \frac{15+14}{36} = \frac{29}{36}$

A9c) $\quad \frac{1}{101} \div \frac{1}{103} \simeq 1 \quad\quad$ note: $\frac{1}{101} \div \frac{1}{101} = 1$

A12) $\quad\quad \frac{5}{8}d = 6000$

$\quad\quad\quad$ #students in dorms

$d = $ # students in college

$\quad\quad\quad\quad\quad d = 6000\left(\frac{8}{5}\right)$

$\quad\quad\quad\quad\quad = 1200(8) = 9600$ students

B14) $\quad$ (a) $100,000\left(\frac{1}{10}\right) = 10,000 \quad\Rightarrow$ make $110,000$

$\quad\quad\quad\quad 100000\left(\frac{11}{10}\right) = 110,000 \quad (1^{st}\ yr)$

$\quad\quad\quad 2^{nd}\ yr: \quad 110000\left(\frac{11}{10}\right) = 11000(11) = 121,000$

$\quad$ (b) $\quad ?\left(\frac{11}{10}\right) = 99,000 \Longrightarrow ? = 99000\left(\frac{10}{11}\right)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = 90000$

$\quad$ (c) $\quad ?\left(\frac{11}{10}\right)\left(\frac{11}{10}\right) = 363000$

$\quad\quad\quad\quad ? = 363000\left(\frac{100}{121}\right) = 300,000$

wkst

7) $\dfrac{3^{-6}}{2^{-1}} \div \dfrac{\frac{2}{3} - \frac{1}{4}}{6^4}$

$= \dfrac{2}{3^6} \cdot \dfrac{6^4}{\frac{2}{3} - \frac{1}{4}} = \dfrac{2}{3^6 \cdot 2} \cdot \dfrac{2^4 \cdot 3^4}{5/12}$

$= \dfrac{2^5 \cdot 12 \cdot 4}{3 \cdot 5}$

$= \dfrac{2^5 \cdot 4}{3 \cdot 5} = \dfrac{2^7}{15} = \boxed{\dfrac{128}{15}}$

$\dfrac{2}{3} - \dfrac{1}{4} = \dfrac{8}{12} - \dfrac{3}{12}$

$= \dfrac{5}{12}$

$6^4 = (2 \cdot 3)^4$

$\quad = 2^4 \cdot 3^4$

$\dfrac{1}{5/12} = 1 \div \dfrac{5}{12}$

$\quad = 1 \cdot \dfrac{12}{5} = \dfrac{12}{5}$

$\dfrac{1}{5/12}\left(\dfrac{12}{12}\right) = \dfrac{12}{5}$

9) $\dfrac{7}{11} \cdot \dfrac{11}{7}\left(\square - \dfrac{15}{2}\right) = \dfrac{1}{4} \cdot \dfrac{7}{11}$

$\square - \dfrac{15}{2} = \dfrac{7}{44}$

$\square = \dfrac{7}{44} + \dfrac{15}{2}\left(\dfrac{22}{22}\right) = \dfrac{7 + 15(22)}{44} = \dfrac{7 + 330}{44}$

$= \boxed{\dfrac{337}{44}}$