6.4 Ratios and Proportions

Ratio-- $\dfrac{a}{b}$ or $a:b$ (read "a to b")

fraction; rate

ex $\$3.40 / \text{gallon}$

Proportion-- an eqn w/ 2 ratios set equal.

ex $\dfrac{2}{5} = \dfrac{x}{10}$

Example:  In class, there is a 3 to 2 ratio of boys to girls.  Write four different ratios.

part to part-- ① $3:2$ (b:g)      ② $2:3$ (g:b)

part to whole-- ③ $3:5$ (b:w)      ④ $2:5$ (g:w)

w (whole class)

Ex 1.  In the last six months, I drove my car for 4460 miles.  If I continue driving my car at this same rate, then how many miles will I have driven after 2.75 years (total)?

$$\frac{4460 \text{ mi}}{6 \text{ mos}} = \frac{4460 \text{ mi}}{0.5 \text{ yr}} \qquad\qquad \frac{4460}{0.5} = \frac{x}{2.75}$$

$$x = \frac{4460(2.75)}{0.5}\left(\frac{2}{2}\right)$$

$$= \frac{4460(5.5)}{1}$$

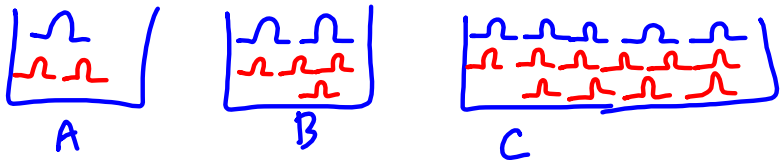$$= \frac{4460(11)}{2} = 2230(11) = 22300 + 2230 = 24{,}530 \text{ mi}$$

Ex 2.  An ad says "3 movies for $18."  At that rate, what is the cost of 5 movies?

① $\dfrac{3 \text{ movies}}{\$18} = \dfrac{5 \text{ movies}}{x}$        ② $\dfrac{\$18}{3 \text{ mov.}} = \dfrac{\$x}{5 \text{ mov}}$

③ $\dfrac{3 \text{ mov.}}{5 \text{ mov.}} = \dfrac{\$18}{\$x}$        ④ $\dfrac{5 \text{ mov}}{3 \text{ mov}} = \dfrac{\$x}{\$18}$

$$x = \$30$$

Ex 3.  In room A, there are 1 blue hat and 2 red hats, in room B, there are 2 blue hats and 4 red hats, in room C, there are 5 blue hats and 10 red hats.  If all the hats in rooms B and C are moved to room A, what will be the ratio of blue hats to red hats?   Can you generalize this?

$$\binom{8 : 16}{(b : r)} = 1 : 2$$

$$b : r = \frac{a_b + b_b + c_b}{a_r + b_r + c_r}$$

Ex 4.  In a photograph of a father and daughter, the daughter's height is 2.3 cm and the father's height is 5.8 cm. If the father is actually 188 cm tall,  how tall is his daughter?

$$\frac{5.8}{188} = \frac{2.3}{x}$$

$$2.3(188) = 5.8x$$

$$\left(\frac{10}{10}\right) \frac{2.3(188)}{5.8} = x$$

$$\frac{8n}{2(3 \cdot 5 \cdot 7)}$$
$$= (3 \cdot 7)(2 \cdot 5)$$

$$x = \frac{23(188)^{94}}{58_{29}} = \frac{23(94)}{29}$$

$$= \frac{2162}{29}$$

(on calculator: 74.551.. ..)

$$\frac{2162}{29} = \boxed{74\frac{16}{29}} \text{ cm} \quad \text{approx}$$

$$2162 - 74(29) = 16$$

Ex 5.  Al is 5 feet tall and has a shadow that is 18 inches long.  At the same time, a tree has a shadow  that is 15 feet long.  Al sets up and solves the proportion as follows:

$$\underset{\text{tree shadow}}{\overset{\text{Al ht}}{\rule{1.5cm}{0.4pt}}} \quad \frac{5\,\text{ft}}{15\,\text{ft}} \times \frac{18\,\text{inches}}{x\,\text{inches}} \quad \underset{\text{tree ht}}{\overset{\text{Al shadow}}{\rule{1.5cm}{0.4pt}}}$$

*proportion is not set up correctly*

Is he correct? **no**  If so, why? If not, how would you help him?

$$\frac{A\,S}{A\,H} \quad \frac{\cancel{18\,in}}{\cancel{5\,ft}} \cancel{=} \frac{\cancel{x\,in}}{\cancel{15\,ft}} \quad \frac{TH}{TS}$$

*the problem is that x should be in feet.*

$$\frac{\cancel{5\,ft}}{\cancel{18\,in}} \cancel{=} \frac{\cancel{x\,ft}}{\cancel{15\,ft}} \quad ①$$

*& convert 18 in into feet*

$$\frac{AN}{AS} \quad \frac{5\,ft}{1.5\,ft} = \frac{x\,ft}{15\,ft} \quad \frac{TH}{TS}$$

② $$\frac{TH}{AH} \quad \frac{x\,ft}{5\,ft} = \frac{15\,ft}{1.5\,ft} \quad \frac{TS}{A\,S}$$

③ $$\frac{1.5}{5} = \frac{15}{x}$$

④ $$\frac{5}{x} = \frac{1.5}{15}$$

fractn ws

#1) $\frac{2}{169} + \frac{33}{104}$

$169 = 13^2$

$104 = 2 \cdot 52 = 2 \cdot 4 \cdot 13$

$LCD = 13 \cdot 13 \cdot 8$

$= \frac{2}{169}\left(\frac{8}{8}\right) + \frac{33}{104}\left(\frac{13}{13}\right)$

$= \frac{16 + 330 + 99}{8 \cdot 13 \cdot 13} = \frac{445}{8 \cdot 13 \cdot 13}$

$13\overline{)445}$
  $\underline{-39}$
   $55$

$= \frac{445}{169(8)} = \boxed{\frac{445}{1352}}$

5) $\frac{13(9)}{(0.9)^2} - \frac{13^2}{10} = \frac{13(9)}{0.9(0.9)}\left(\frac{100}{100}\right) - \frac{13^2}{10}$

$0.9(10)$
$= \frac{9}{10}(10)$
$= 9$

$= \frac{13(9)(100)}{9(9)} - \frac{13^2}{10} = \frac{1300}{9} - \frac{169}{10}$

$= \frac{1300}{9}\left(\frac{10}{10}\right) - \frac{169}{10}\left(\frac{9}{9}\right) = \frac{13000 - 169(9)}{90}$

$= \frac{13000 - 1521}{90} = \boxed{\frac{11479}{90}}$

6.4A
8) (a) $x:y = 3:4$    $\frac{x}{y} = \frac{3}{4}$

$x + y = 98$

$x = \frac{3}{4}y$

3:4
6:8
9:12
12:16
15:20
18:24
21:28

$\frac{3}{4}y + y = 98$

$\frac{4}{7} \cdot \frac{7}{4}y = 98 \cdot \frac{4}{7} = 14.4 = 56$

$y = 56 \Rightarrow x = 42$

$\frac{3}{7} \cdot \frac{98}{1} = ?$    $\frac{3}{7} = \frac{?}{98}$    $? = 42$

(b) product is 768    $xy = 768$    $\frac{3}{4} = \frac{x}{y}$

$2^8 = 256$    $\frac{3}{4}(y \cdot y) = 768$    $x = \frac{3}{4}y$

$\sqrt{256} = \sqrt{2^8} = 2^4$    $y^2 = \cancel{768} \cdot \frac{4}{3} = 256(4)$    (256 above cancel)

① $y = 32, \quad x = \frac{3}{4}(32) = 24$    $y = \pm 16(2)$

② $y = -32, \quad x = -24$    $= \pm 32$

A9)    2:3:5    total $82 000

G:B:C

G: $\frac{2}{10}(82,000) = \$16,400$

B: $\frac{3}{10}(82000) = \$24,600$

C: $\frac{5}{10}(82000) = \$41,000$

B21) $\frac{a}{b} = \frac{c}{d}$ , $a \neq -b$, $a \neq b$

(a) prove $\frac{a+b}{b} = \frac{c+d}{d}$

Pf we know

$\frac{a}{b} = \frac{c}{d} \implies \frac{a}{b} + 1 = \frac{c}{d} + 1$

$\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$

$\frac{a+b}{b} = \frac{c+d}{d}$ ✓

(b) prove: $\frac{a}{a+b} = \frac{c}{c+d}$

Pf we know $\frac{a}{b} = \frac{c}{d}$.

$\implies \frac{ad}{bd} = \frac{bc}{bd}$     (mult. by 1)

$\implies ad = bc$     (mult by bd on both sides)

$\implies ac + ad = ac + bc$     (add ac to both sides)

$\implies a(c+d) = c(a+b)$     (factor) both sides

$\implies \frac{a\cancel{(c+d)}}{\cancel{(c+d)}(a+b)} = \frac{c\cancel{(a+b)}}{(c+d)\cancel{(a+b)}}$     (divide both sides by $(c+d)(a+b)$)

$\implies \frac{a}{a+b} = \frac{c}{c+d}$     (simplify) ✓

A10)  S  $3\frac{1}{2}$ hr

D  $4\frac{1}{2}$ hr

total pay $176

$3\frac{1}{2} : 4\frac{1}{2} = 3.5 : 4.5 = 35 : 45 = 7 : 9$

S pay $= \frac{7}{16}(176) = \$77$

D pay $= \frac{9}{16}(176) = \$99$