7.2/7.3 Operations on Decimals/Nonterminating Decimals

There are three types of decimals.

1. Terminating (ℚ) ex -1.52

   ends/terminates

2. Repeating, non-terminating (ℚ) ex $0.\overline{3}$, $0.17\overline{231}$

   never ends, but repeats forever

3. Non-repeating, non-terminating (irrational)

   ex π, $\sqrt{\pi}$, $\sqrt{2}$, $\sqrt{5}$

   0.10110111011111....

Which of these types are rational numbers? (1) and (2)

For the type(s) of decimals that are not rational, what do we call those numbers? irrational

Arithmetic with decimals

Examples:

1. 1.36475002 + 0.0007819

$$\begin{array}{r} 1.36475002 \\ +\ 0.0007819 \\ \hline 1.36553192 \end{array}$$

2. 1.36475002 - 0.0007819

$$\begin{array}{r} 1.36475002 \\ -\ 0.0007819 \\ \hline 1.36396812 \end{array}$$

3. 362.14 (4.3) $= \left[\frac{36214}{100}(43)\right]\left[\frac{43}{10}\right] = [36214(43)]\left(\frac{1}{100(10)}\right)$

$= 36214(43)(0.001)$

$= 1557.202$

4. 129.31 ÷ 3.2 $= \left(\frac{12931}{100} \div \frac{32}{10}\right) = \frac{12931}{100} \cdot \frac{10}{32} = \frac{12931}{32}\left(\frac{1}{10}\right)$

$= 40.409375$

$$\begin{array}{r} 404.09375 \\ 32\overline{)\,12931} \\ -128 \\ \hline 131 \\ -128 \\ \hline 300 \\ -288 \\ \hline 120 \\ -96 \\ \hline 240 \\ -224 \\ \hline 160 \end{array}$$

or $3.2\overline{)\,129.31}$

$\frac{129.31}{3.2}\left(\frac{10}{10}\right) = \frac{1293.1}{32}$

Express 0.111.... as a fraction.

$n = 0.111\ldots$

$10n = 1.111\ldots$

$-\ n = 0.111\ldots$

$9n = 1 \Rightarrow n = 1/9$

$0.333\ldots = 0.111\ldots(3)$

$\frac{1}{3} = 0.111\ldots(3)$

$\frac{1}{9} = 0.111\ldots$

How about 0.22222... ? 0.33333....? 0.4444....?

$n = 0.222\ldots$

$10n = 2.222\ldots$

$10n = 2.22\ldots$

$-\ n = 0.22\ldots$

$9n = 2$

$n = 2/9$

Then, what's 0.9999.....?

$0.99\ldots = 1$

$10n = 9.99\ldots$

$-\ n = 0.99\ldots$

$9n = 9$

$n = 1$

Can we express 0.51515151..... as a fraction? If so, what is its fraction form?

$n = 0.\overline{51} = 51/99$

$100n = 51.\overline{51}$

$-\ n = 0.\overline{51}$

$99n = 51$

$n = 51/99$

$0.\overline{63} = 63/99 = \frac{21}{33} = \frac{7}{11}$

$0.\overline{12} = 12/99 = \frac{4}{33}$

More Examples:

Convert these repeating decimals to fractions (notice patterns).

5. 0.272727.... $= n = \frac{27}{99}$

$0.000\overline{27} = \frac{27}{99000}$

6. 0.027272727.... $= n = \frac{27}{990}$

$0.00\overline{27} = \frac{27}{9900}$

$1000n = 27.\overline{27}$
$-\ 10n = 0.\overline{27}$
$990n = 27$

$0.00\overline{27} = \frac{27}{99,000}$

$0.0\overline{61} = \frac{61}{990}$

7. 0.002727272727.... $= \frac{27}{9900}$

$0.000\overline{61} = \frac{61}{99,000}$

$10000n = 27.\overline{27}$
$-\ 100n = 0.\overline{27}$
$9900n = 27$

8. 0.527272727.... $= n$ $\quad n = \frac{522}{990}$

$1000n = 527.\overline{27}$
$-\ 10n = 5.\overline{27}$
$990n = 522$

9. 0.327272727.... $= \frac{327-3}{990} = \frac{324}{990}$

$0.12727\ldots = \frac{126}{990}$

$0.92727\ldots = \frac{927-9}{990} = \frac{918}{990}$

$0.0\overline{35} = \frac{35}{990}$ $\quad 0.7\overline{35} = \frac{728}{990}$

$0.2\overline{35} = \frac{233}{990}$

Scientific Notation $y \times 10^b$, $1 \le y < 10$, $b \in \mathbb{Z}$ (or $-10 < y \le -1$)

Express these decimals in scientific notation.

(a) 5678.0021

$= 5.67800021 \times 10^3$

(b) -0.00000000962 $= -9.62 \times 10^{-9}$

Express these numbers, given in scientific notation, as decimals.

(a) 3.456009×10^9 $= 3,456,009,000$

(b) -8.7765×10^{-4} $= -0.00087765$

Order these decimals from smallest to largest.

B $5.161616\ldots$
C $5.166666\ldots$
D $5.161616\ldots$

-5.16, $-5.\overline{16}$, $-5.1\overline{6}$, $-5.16\overline{1}$, $-5.6\overline{16}$

A B C D
$5.16, 5.\overline{16}, 5.1\overline{6}, 5.1\overline{61},$
$5.6\overline{16}$
✓ ✓ E

$-5.6\overline{16} < -5.1\overline{6} < -5.16\overline{1}$

$= -5.\overline{16} < -5.16$

$5.16 < 5.\overline{16} = 5.16\overline{1} < 5.1\overline{6} < 5.6\overline{16}$

7.2A

3) 25 lb @ peaches $4.00/lb, 15 lb @ apples $2.00/lb

10 lbs @ $x/lb to have avg $/lb of $3.50/lb

$$\frac{25(4)+15(2)+10x}{50}=3.5$$

$$\frac{130+10x}{50}=3.5$$

$$130+10x=35(5)$$

$$130+10x=175$$

$$10x=45$$

$$x=4.5$$

$4.50/lb

A24) 49,736.528 1

B13) purchased: 964 + 27,422.50 + 495 = 28,881.50

sold: 18(61.48) + 85.35(350) = 30,889.14

profit: 30889.14 − 28881.50

=

price per share?

$$\frac{27422.5}{350}=78.35$$

7.3

A11) halfway between $0.\overline{4}$ and 0.5

$$\frac{0.\overline{4}+0.5}{2} = \frac{4/9+1/2}{2} = \frac{\frac{8+9}{18}}{2} = \frac{17}{18}\div 2$$

$$= \frac{17}{18}\cdot\frac{1}{2} = \frac{17}{36}$$

$$= \frac{1}{4}\left(\frac{17}{9}\right) = \frac{1}{4}\left(1\frac{8}{9}\right)$$

$$= 0.25\left(1.\overline{8}\right)$$

$$\frac{0.\overline{4}+0.5}{2} = \frac{0.9\overline{4}}{2} = 0.4\overline{72}$$

B3) (a) 21st digit in $\frac{3}{7}$

$$7\overline{)3.0000000} = .428571\overline{4}$$

-28, 20, -14, 60, -56, 40, -35, 50, -49, 10, -7, 30

$\frac{3}{7} = 0.\overline{428571}$

21st digit is 8

105th " " 5

200th " " 2

5280th " " 1

A14) $-1.45\overline{4}, -1.4\overline{54}, -1.\overline{454}, -1.45\overline{4}, -1.454$

$\Leftrightarrow$ $-1.4545454\ldots, -1.4544544544\ldots, -1.4545445\ldots, -1.4544444\ldots, -1.454000\ldots$

$$-1.454 > -1.45444\ldots > -1.4544544544\ldots > -1.4545454\ldots$$

$$= -1.4545454\ldots$$

$$-1.454 > -1.45\overline{4} > -1.4\overline{54} > -1.\overline{454} = -1.\overline{45}$$

B2) (e) $-4.3\overline{4} = -(4+0.3\overline{4}) = -\left(4\frac{31}{90}\right) = -4\frac{31}{90}$

$n = 0.3\overline{4}$

$100n = 34.\overline{4}$

$-10n = 3.\overline{4}$

$90n = 31$

$n = 31/90$

(f) $-0.0\overline{3} = -\frac{1}{30}$

$n = 0.0\overline{3}$

$100n = 3.\overline{3}$

$-10n = 0.\overline{3}$

$90n = 3$

$n = \frac{3}{90} = \frac{1}{30}$