

7.2/7.3 Operations on Decimals/Nonterminating Decimals

There are three types of decimals.

1. Terminating (\mathbb{Q}) ex -1.52

ends/terminates

2. Repeating, non-terminating (\mathbb{Q}) ex $0.\bar{3}$, $0.17\overline{231}$

never ends, but repeats forever

3. Non-repeating, non-terminating (irrational)
ex π , $\sqrt{\pi}$, $\sqrt{2}$, $\sqrt{5}$

$0.1011011101111\dots$

Which of these types are rational numbers? (1) and (2)

For the type(s) of decimals that are not rational, what do we call those numbers? irrational

Arithmetic with decimals

Examples:

1. $1.36475002 + 0.0007819$

$$\begin{array}{r} 1.36475002 \\ + 0.0007819 \\ \hline 1.36553192 \end{array}$$

2. $1.36475002 - 0.0007819$

$$\begin{array}{r} 1.36475002 \\ - 0.0007819 \\ \hline 1.36396812 \end{array}$$

$$\begin{aligned} 3. \quad 362.14(4.3) &= \left[\frac{36214}{100} \left(\frac{43}{10} \right) \right] = [36214(43)] \left(\frac{1}{100(10)} \right) \\ &= 36214(43)(0.001) \\ &= 1557.202 \end{aligned}$$

$$\begin{aligned} 4. \quad 129.31 \div 3.2 &= \left(\frac{12931}{100} \div \frac{32}{10} \right) = \frac{12931}{100} \cdot \frac{10}{32} = \frac{12931}{32} \left(\frac{1}{10} \right) \\ &= 40.409375 \end{aligned}$$

$$\begin{array}{r} 404.09375 \\ 32 \overline{) 12931} \\ \underline{-128} \\ 131 \\ \underline{-128} \\ 300 \\ \underline{-288} \\ 120 \\ \underline{-96} \\ 240 \\ \underline{-224} \\ 160 \end{array}$$

$$\text{or } 3.2 \overline{) 129.31}$$

$$\frac{129.31}{3.2} \left(\frac{10}{10} \right) = \frac{1293.1}{32}$$

Express $0.111\dots$ as a fraction.

$$\begin{aligned} n &= 0.111\dots \\ 10n &= 1.111\dots \\ - n &= 0.111\dots \\ \hline 9n &= 1 \Rightarrow n = 1/9 \end{aligned}$$

$$\begin{aligned} 0.333\dots &= 0.111\dots (3) \\ \frac{1}{3} &= 0.111\dots (3) \\ \frac{1}{9} &= 0.111\dots \end{aligned}$$

How about $0.2222\dots$? $0.33333\dots$? $0.4444\dots$?

$$\begin{aligned} n &= 0.222\dots \\ 10n &= 2.222\dots \end{aligned}$$

$$\begin{aligned} 10n &= 2.22\dots \\ - n &= 0.22\dots \\ \hline 9n &= 2 \end{aligned}$$

Then, what's $0.9999\dots$?

$$0.99\dots = 1$$

$$\begin{aligned} 10n &= 9.99\dots \\ - n &= 0.999\dots \\ \hline 9n &= 9 \\ n &= 1 \end{aligned}$$

Can we express $0.515151\dots$ as a fraction? If so, what is its fraction form?

$$\begin{aligned} n &= 0.\overline{51} = 51/99 \\ 100n &= 51.\overline{51} \\ - n &= 0.\overline{51} \\ \hline 99n &= 51 \\ n &= 51/99 \end{aligned}$$

$$\begin{aligned} 0.\overline{63} &= 63/99 = \frac{21}{33} \\ &= \frac{7}{11} \\ 0.\overline{12} &= 12/99 = \frac{4}{33} \end{aligned}$$

More Examples:

Convert these repeating decimals to fractions (notice patterns).

$$5. \ 0.272727\dots = n = \frac{27}{99}$$

$$6. \ 0.027272727\dots = n = \frac{27}{990}$$

$$\begin{array}{r} 1000n = 27.\overline{27} \\ - 10n = 0.\overline{27} \\ \hline 990n = 27 \end{array}$$

$$7. \ 0.002727272727\dots = \frac{27}{9900}$$

$$\begin{array}{r} 10000n = 27.\overline{27} \\ - 100n = 0.\overline{27} \\ \hline 9900n = 27 \end{array}$$

$$8. \ 0.527272727\dots = n$$

$$n = \frac{522}{990}$$

$$\begin{array}{r} 1000n = 527.\overline{27} \\ - 10n = 5.\overline{27} \\ \hline 990n = 522 \end{array}$$

$$9. \ 0.327272727\dots = \frac{327-3}{990} = \frac{324}{990}$$

$$0.92727\dots = \frac{927-9}{990} = \frac{918}{990}$$

$$0.000\overline{27} = \frac{27}{99000}$$

$$0.000\overline{27} = \frac{27}{99,000}$$

$$0.0\overline{61} = \frac{61}{990}$$

$$0.000\overline{61} = \frac{61}{99,000}$$

$$\begin{aligned} 0.12727\dots \\ = \frac{126}{990} \end{aligned}$$

$$0.0\overline{35} = \frac{35}{990}$$

$$0.7\overline{35} = \frac{728}{990}$$

$$0.2\overline{35} = \frac{233}{990}$$

Scientific Notation

$$y \times 10^b, \quad 1 \leq y < 10, \quad b \in \mathbb{Z}$$

(or $-10 < y \leq -1$)

Express these decimals in scientific notation.

(a) 5678.0021

$$= 5.6780021 \times 10^3$$

(b) -0.00000000962 = -9.62×10^{-9}

Express these numbers, given in scientific notation, as decimals.

(a) $3.456009 \times 10^9 = 3,456,009,000$

(b) $-8.7765 \times 10^{-4} = -0.00087765$

Order these decimals from smallest to largest.

$-5.1\overline{6}$, $-5.\overline{16}$, $-5.\overline{16}$, $-5.\overline{161}$, $-5.\overline{616}$

$$-5.\overline{616} < -5.\overline{16} < -5.\overline{161}$$

$$= -5.\overline{16} < -5.1\overline{6}$$

B $5.1\overline{6161616}\dots$

C $5.1\overline{6666}\dots$

D $5.1\overline{616161}\dots$

A	B	C	D
$5.1\overline{6}$	$5.\overline{16}$	$5.\overline{16}$	$5.\overline{161}$
✓			
		$5.\overline{616}$	
		✓	

$$5.1\overline{6} < 5.\overline{16} = 5.\overline{161} < 5.\overline{16} < 5.\overline{616}$$

7.2A

3) 25 lb @ ^{peaches} \$4.00/lb, 15 lb @ ^{apples} \$2.00/lb

10 lbs @ \$x/lb to have avg \$/lb of \$3.50/lb

$$\frac{25(4) + 15(2) + 10x}{50} = 3.5$$

$$\frac{130 + 10x}{50} = 3.5$$

$$\boxed{\$4.50/\text{lb}}$$

$$130 + 10x = 35(5)$$

$$130 + 10x = 175$$

$$10x = 45$$

$$x = 4.5$$

A24)

$$4 \underline{9}, \underline{7} \underline{3} \underline{6} . \underline{5} \underline{2} \underline{8} \underline{1}$$

~~128679~~
B13)

purchased: $964 + 27,422.50$
 $+ 495 = 28,881.50$

sold: $18(61.48) + 85.35(350)$
 $= 30,889.14$

profit: $30,889.14 - 28,881.50$

=

price per share?

$$\frac{27422.5}{350}$$

$$= 78.35$$

7.3 halfway
A11) $0.\overline{4}$ and 0.5

$$\frac{0.\overline{4} + 0.5}{2} = \frac{\frac{4}{9} + \frac{1}{2}}{2} = \frac{\frac{8+9}{18}}{2} = \frac{17}{18} \div 2$$

$$\frac{0.\overline{4} + 0.5}{2} = \frac{0.9\overline{4}}{2} = 0.47\overline{2}$$

$$\frac{17}{18} \cdot \frac{1}{2} = \frac{17}{36}$$

$$= \frac{1}{4} \left(\frac{17}{9} \right) = \frac{1}{4} \left(1.\overline{8} \right) = 0.25(1.\overline{8})$$

B3) (a) 21st digit in $\frac{3}{7}$

$$\begin{array}{r} 0.4285714 \\ 7 \overline{) 3.0000000} \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-35} \\ 50 \\ \underline{-49} \\ 10 \\ \underline{-7} \\ 30 \\ \underline{-21} \\ 90 \\ \underline{-63} \\ 270 \\ \underline{-210} \\ 60 \\ \underline{-42} \\ 180 \\ \underline{-126} \\ 540 \\ \underline{-378} \\ 1620 \\ \underline{-1134} \\ 4860 \\ \underline{-3402} \\ 14580 \\ \underline{-10206} \\ 43740 \\ \underline{-30618} \\ 131220 \\ \underline{-91854} \\ 393660 \\ \underline{-275562} \\ 1180980 \\ \underline{-826686} \\ 3542940 \\ \underline{-2479858} \\ 10630820 \\ \underline{-7441574} \\ 31892440 \\ \underline{-22324708} \\ 95677332 \\ \underline{-67974132} \\ 277032000 \\ \underline{-193922400} \\ 831096000 \\ \underline{-581767200} \\ 2493288000 \\ \underline{-1745301600} \\ 7487866400 \\ \underline{-5241506480} \\ 22463599200 \\ \underline{-15724519440} \\ 67410479760 \\ \underline{-47187335832} \\ 202231439272 \\ \underline{-141562007490} \\ 606694317782 \\ \underline{-424666022446} \\ 1820282953336 \\ \underline{-1274198067134} \\ 5460841466202 \\ \underline{-3822589026341} \\ 1638252539861 \\ \underline{-1146776777902} \\ 491475761959 \\ \underline{-344033033371} \\ 147442458588 \\ \underline{-103209721011} \\ 44232737477 \\ \underline{-30962916230} \\ 13269821247 \\ \underline{-9286874872} \\ 4001133759 \\ \underline{-2800793631} \\ 1200340128 \\ \underline{-840240089} \\ 3600680239 \\ \underline{-2520476167} \\ 1080203672 \\ \underline{-756142160} \\ 3240561512 \\ \underline{-2268393058} \\ 9721682054 \\ \underline{-6805177438} \\ 2916504616 \\ \underline{-2041553231} \\ 8124491385 \\ \underline{-5687144069} \\ 2437347316 \\ \underline{-1706143121} \\ 7667334195 \\ \underline{-5367133937} \\ 2300200258 \\ \underline{-1610140180} \\ 6391860078 \\ \underline{-4474302054} \\ 1944558073 \\ \underline{-1361190631} \\ 5784390442 \\ \underline{-4049073309} \\ 1735313133 \\ \underline{-1214719193} \\ 5148412140 \\ \underline{-3593888500} \\ 1589023290 \\ \underline{-1113376283} \\ 4776846607 \\ \underline{-3343792625} \\ 1443067332 \\ \underline{-1010207102} \\ 4326860230 \\ \underline{-3028802161} \\ 1298058069 \\ \underline{-908641648} \\ 3879416421 \\ \underline{-2715591497} \\ 1163814924 \\ \underline{-814670267} \\ 3524144657 \\ \underline{-2466901260} \\ 1077243397 \\ \underline{-751870461} \\ 3222273236 \\ \underline{-2255591262} \\ 966714074 \\ \underline{-676601892} \\ 2890538852 \\ \underline{-2023377196} \\ 8881911656 \\ \underline{-6217138159} \\ 2664797847 \\ \underline{-1865358493} \\ 7809439354 \\ \underline{-5487607648} \\ 2321831606 \\ \underline{-1635282124} \\ 6584149482 \\ \underline{-4608914637} \\ 1975234845 \\ \underline{-1392664391} \\ 5362480454 \\ \underline{-3753737118} \\ 1608743336 \\ \underline{-1131430316} \\ 4757303020 \\ \underline{-3330112114} \\ 1427190806 \\ \underline{-979927124} \\ 4297270682 \\ \underline{-3009089478} \\ 1288361734 \\ \underline{-882063216} \\ 4015153518 \\ \underline{-2811421261} \\ 1203732257 \\ \underline{-842612580} \\ 3612540677 \\ \underline{-2528778274} \\ 1083762403 \\ \underline{-756067646} \\ 3281674757 \\ \underline{-2317172330} \\ 9649575227 \\ \underline{-6754703059} \\ 2874862168 \\ \underline{-1992403518} \\ 8756218150 \\ \underline{-6129552711} \\ 2643662439 \\ \underline{-1830563707} \\ 8606058732 \\ \underline{-6024241312} \\ 2581834600 \\ \underline{-1797284220} \\ 8018762380 \\ \underline{-5593135466} \\ 2425626914 \\ \underline{-1697938840} \\ 7558728074 \\ \underline{-5291110651} \\ 2229617019 \\ \underline{-1560791913} \\ 6737875106 \\ \underline{-4676512573} \\ 2070363833 \\ \underline{-1449254683} \\ 6284383150 \\ \underline{-4379068205} \\ 1905316945 \\ \underline{-1337471861} \\ 5717895084 \\ \underline{-3982527559} \\ 1719367329 \\ \underline{-1193559130} \\ 5524314159 \\ \underline{-3867020901} \\ 1637292068 \\ \underline{-1127108459} \\ 5197205709 \\ \underline{-3638044056} \\ 1529161253 \\ \underline{-1070522877} \\ 4558638376 \\ \underline{-3192306463} \\ 1366331913 \\ \underline{-956802521} \\ 4111535392 \\ \underline{-2888075774} \\ 1223457816 \\ \underline{-850440766} \\ 3764117050 \\ \underline{-2635282935} \\ 1108834115 \\ \underline{-760364020} \\ 3428470095 \\ \underline{-2380929266} \\ 1047540169 \\ \underline{-730518447} \\ 3147951722 \\ \underline{-2173566205} \\ 9303851017 \\ \underline{-6592695701} \\ 2711181316 \\ \underline{-784585370} \\ 2026625946 \\ \underline{-2171992321} \\ 8094267145 \\ \underline{-5666987001} \\ 2427269144 \\ \underline{-1658154512} \\ 8616106632 \\ \underline{-2462412368} \\ 6153694264 \\ \underline{-1130121700} \\ 5023572564 \\ \underline{-3390850544} \\ 1632722020 \\ \underline{-994421406} \\ 6332700614 \\ \underline{-4472950450} \\ 1859750164 \\ \underline{-1299725112} \\ 6293025552 \\ \underline{-3813617774} \\ 2479407778 \\ \underline{-1078202160} \\ 1401205612 \\ \underline{-338057480} \\ 1063148132 \\ \underline{-905006592} \\ 1528141540 \\ \underline{-2377894272} \\ 3703521268 \\ \underline{-1053005856} \\ 2650515412 \\ \underline{-687603392} \\ 1962912020 \\ \underline{-1840028448} \\ 1122883572 \\ \underline{-4696212192} \\ 3505747080 \\ \underline{-1261509072} \\ 2244238008 \\ \underline{-612362736} \\ 1631875272 \\ \underline{-152016192} \\ 1479859080 \\ \underline{-382414720} \\ 1097444360 \\ \underline{-956055680} \\ 1421388680 \\ \underline{-236847168} \\ 1184541512 \\ \underline{-591112384} \\ 593429124 \\ \underline{-147857280} \\ 445571844 \\ \underline{-369384704} \\ 786187140 \\ \underline{-92048032} \\ 694139108 \\ \underline{-230116832} \\ 464022276 \\ \underline{-57504168} \\ 406518108 \\ \underline{-14376162} \\ 392141946 \\ \underline{-35940416} \\ 356201530 \\ \underline{-8975112} \\ 347226418 \\ \underline{-22437632} \\ 324788786 \\ \underline{-5614664} \\ 319174122 \\ \underline{-1403666} \\ 317770456 \\ \underline{-3509166} \\ 314261290 \\ \underline{-877291} \\ 313384000 \\ \underline{-2193102} \\ 311190898 \\ \underline{-5477764} \\ 305713134 \\ \underline{-1369341} \\ 304343793 \\ \underline{-3423353} \\ 300920440 \\ \underline{-8558382} \\ 291362052 \\ \underline{-21395504} \\ 269966548 \\ \underline{-5348876} \\ 264617672 \\ \underline{-13371691} \\ 251245981 \\ \underline{-33429227} \\ 217816754 \\ \underline{-83573168} \\ 134243586 \\ \underline{-20810942} \\ 113432644 \\ \underline{-52075606} \\ 61357038 \\ \underline{-15519252} \\ 45837786 \\ \underline{-3884916} \\ 41952870 \\ \underline{-9462290} \\ 32490580 \\ \underline{-2367073} \\ 29823507 \\ \underline{-5955926} \\ 23867581 \\ \underline{-1488941} \\ 22378640 \\ \underline{-3717352} \\ 18661288 \\ \underline{-929338} \\ 17731950 \\ \underline{-2322976} \\ 15408974 \\ \underline{-5807441} \\ 9601533 \\ \underline{-1451861} \\ 8149672 \\ \underline{-3629652} \\ 4520020 \\ \underline{-907263} \\ 3612757 \\ \underline{-2268164} \\ 1344593 \\ \underline{-566141} \\ 778452 \\ \underline{-142035} \\ 636417 \\ \underline{-35609} \\ 600808 \\ \underline{-89152} \\ 511656 \\ \underline{-22288} \\ 489368 \\ \underline{-5561} \\ 483807 \\ \underline{-1391} \\ 482416 \\ \underline{-347} \\ 482069 \\ \underline{-86} \\ 481983 \\ \underline{-21} \\ 481962 \\ \underline{-5} \\ 481957 \\ \underline{-1} \\ 481956 \\ \underline{-1} \\ 481955 \\ \underline{-1} \\ 481954 \\ \underline{-1} \\ 481953 \\ \underline{-1} \\ 481952 \\ \underline{-1} \\ 481951 \\ \underline{-1} \\ 481950 \\ \underline{-1} \\ 481949 \\ \underline{-1} \\ 481948 \\ \underline{-1} \\ 481947 \\ \underline{-1} \\ 481946 \\ \underline{-1} \\ 481945 \\ \underline{-1} \\ 481944 \\ \underline{-1} \\ 481943 \\ \underline{-1} \\ 481942 \\ \underline{-1} \\ 481941 \\ \underline{-1} \\ 481940 \\ \underline{-1} \\ 481939 \\ \underline{-1} \\ 481938 \\ \underline{-1} \\ 481937 \\ \underline{-1} \\ 481936 \\ \underline{-1} \\ 481935 \\ \underline{-1} \\ 481934 \\ \underline{-1} \\ 481933 \\ \underline{-1} \\ 481932 \\ \underline{-1} \\ 481931 \\ \underline{-1} \\ 481930 \\ \underline{-1} \\ 481929 \\ \underline{-1} \\ 481928 \\ \underline{-1} \\ 481927 \\ \underline{-1} \\ 481926 \\ \underline{-1} \\ 481925 \\ \underline{-1} \\ 481924 \\ \underline{-1} \\ 481923 \\ \underline{-1} \\ 481922 \\ \underline{-1} \\ 481921 \\ \underline{-1} \\ 481920 \\ \underline{-1} \\ 481919 \\ \underline{-1} \\ 481918 \\ \underline{-1} \\ 481917 \\ \underline{-1} \\ 481916 \\ \underline{-1} \\ 481915 \\ \underline{-1} \\ 481914 \\ \underline{-1} \\ 481913 \\ \underline{-1} \\ 481912 \\ \underline{-1} \\ 481911 \\ \underline{-1} \\ 481910 \\ \underline{-1} \\ 481909 \\ \underline{-1} \\ 481908 \\ \underline{-1} \\ 481907 \\ \underline{-1} \\ 481906 \\ \underline{-1} \\ 481905 \\ \underline{-1} \\ 481904 \\ \underline{-1} \\ 481903 \\ \underline{-1} \\ 481902 \\ \underline{-1} \\ 481901 \\ \underline{-1} \\ 481900 \\ \underline{-1} \\ 481899 \\ \underline{-1} \\ 481898 \\ \underline{-1} \\ 481897 \\ \underline{-1} \\ 481896 \\ \underline{-1} \\ 481895 \\ \underline{-1} \\ 481894 \\ \underline{-1} \\ 481893 \\ \underline{-1} \\ 481892 \\ \underline{-1} \\ 481891 \\ \underline{-1} \\ 481890 \\ \underline{-1} \\ 481889 \\ \underline{-1} \\ 481888 \\ \underline{-1} \\ 481887 \\ \underline{-1} \\ 481886 \\ \underline{-1} \\ 481885 \\ \underline{-1} \\ 481884 \\ \underline{-1} \\ 481883 \\ \underline{-1} \\ 481882 \\ \underline{-1} \\ 481881 \\ \underline{-1} \\ 481880 \\ \underline{-1} \\ 481879 \\ \underline{-1} \\ 481878 \\ \underline{-1} \\ 481877 \\ \underline{-1} \\ 481876 \\ \underline{-1} \\ 481875 \\ \underline{-1} \\ 481874 \\ \underline{-1} \\ 481873 \\ \underline{-1} \\ 481872 \\ \underline{-1} \\ 481871 \\ \underline{-1} \\ 481870 \\ \underline{-1} \\ 481869 \\ \underline{-1} \\ 481868 \\ \underline{-1} \\ 481867 \\ \underline{-1} \\ 481866 \\ \underline{-1} \\ 481865 \\ \underline{-1} \\ 481864 \\ \underline{-1} \\ 481863 \\ \underline{-1} \\ 481862 \\ \underline{-1} \\ 481861 \\ \underline{-1} \\ 481860 \\ \underline{-1} \\ 481859 \\ \underline{-1} \\ 481858 \\ \underline{-1} \\ 481857 \\ \underline{-1} \\ 481856 \\ \underline{-1} \\ 481855 \\ \underline{-1} \\ 481854 \\ \underline{-1} \\ 481853 \\ \underline{-1} \\ 481852 \\ \underline{-1} \\ 481851 \\ \underline{-1} \\ 481850 \\ \underline{-1} \\ 481849 \\ \underline{-1} \\ 481848 \\ \underline{-1} \\ 481847 \\ \underline{-1} \\ 481846 \\ \underline{-1} \\ 481845 \\ \underline{-1} \\ 481844 \\ \underline{-1} \\ 481843 \\ \underline{-1} \\ 481842 \\ \underline{-1} \\ 481841 \\ \underline{-1} \\ 481840 \\ \underline{-1} \\ 481839 \\ \underline{-1} \\ 481838 \\ \underline{-1} \\ 481837 \\ \underline{-1} \\$$