8.1 Real Numbers

$\mathbb{R}$

The set of Real Numbers, denoted by **R**, is the union of the rational numbers and irrational numbers.

In decimal form, irrational numbers are non-terminating and non-repeating.

Examples:

$\pm\pi$, $\pm\sqrt{2}$, $\pm\sqrt{5}$, $\pm\sqrt{\pi}$, $\pm\sqrt{\text{any prime \#}}$, $\sqrt[3]{\text{any prime \#}}$,

$\pm\sqrt[4]{5}$, $-1.2020020002000 0\ldots$

Properties of Real Numbers (for addition and multiplication)

1. Closure

(closed also under subtraction)

2. Commutativity & Associativity ✓

$a+b=b+a$ | $a+(b+c)=(a+b)+c$

$ab=ba$ | $a(bc)=(ab)c$

3. Distributivity

$a(b\pm c)=ab\pm ac$

4. Identities

1, 0 are in this set

5. Inverses

if $a\in\mathbb{R}$, then $\frac{1}{a}$ and $-a\in\mathbb{R}$, where $\frac{1}{a}$ makes sense.

6. Denseness

between any two $\mathbb{R}$ #s, there are uncountably infinitely many other $\mathbb{R}$ #s.

What properties does the set of irrational numbers have?

Irrational #s

1. not closed for any arithmetic operation

2. commutativity & associativity for mult. and addition

3. Distributivity

Prove that there are infinitely many primes.

(Proof by contradiction)

Pf Assume there are finitely many primes.
Let's say there are n of them. Call them

$$p_1, p_2, \ldots, p_{n-1}, p_n$$

Then create this number

$p = p_1 \cdot p_2 \cdot p_3 \cdots p_n + 1$. Notice $p > p_n$.

Since p_n is the biggest prime # and $p > p_n$, then p is composite.

Then we should be able to factor p into prime factors.

But $p_1 \cdot p_2 \cdots p_n$ and 1 have no common prime factors $\Rightarrow$ we have a contradiction.

$\Rightarrow$ our original assumptn is wrong

$\Rightarrow$ there are infinitely many primes. //

Prove that $\sqrt{2}$ is irrational.

Pf (Pf by contradiction)

Assume $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{a}{b}$ for some $a, b \in \mathbb{W}$, $b \neq 0$;

$GCF(a,b) = 1$. $\sqrt{2} = \frac{a}{b} \iff \sqrt{2}\, b = a$

$\iff 2b^2 = a^2$

Use Fundamental Thm of Arithmetic.

So, the prime factorization of $2b^2$ and a^2 is the same.

but a^2 has an even # of prime factors

and $2b^2$ " " odd " " " "

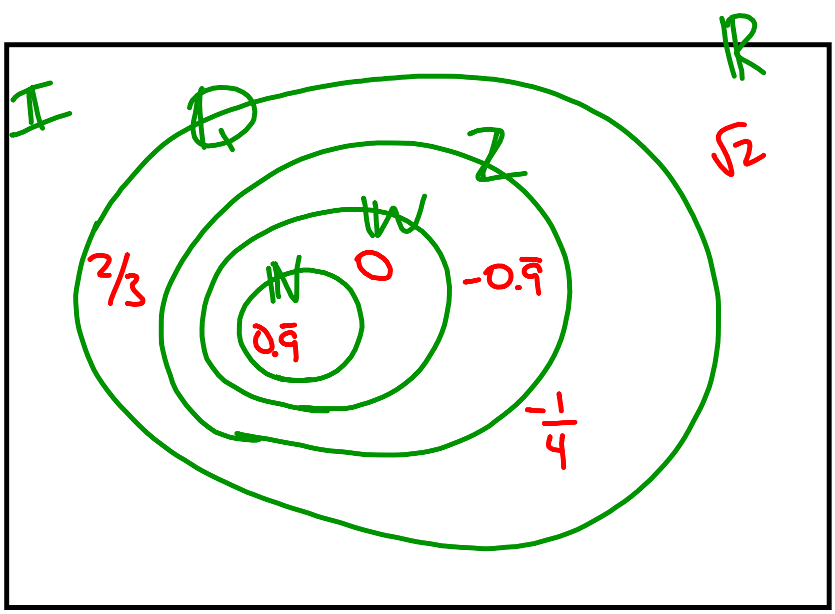
$\Rightarrow 2b^2$ cannot equal $a^2 \Rightarrow$ we have a contradictn

$\Rightarrow \sqrt{2}$ is irrational.

Draw Venn Diagram for all the sets of numbers considered this semester (**N**, **W**, **Z**, **Q**, **R** and irrationals (**I**)).

$\mathbb{N} \to \mathbb{W} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R}$

$\mathbb{I} \to \mathbb{R}$



$\sqrt{2}$

$0.\overline{9}$

$-0.\overline{9}$

$2/3$ $-1/4$

0

Fractional Exponents

ex $x^3 = 8$

$(x^3)^{1/3} = 8^{1/3}$

$x = 8^{1/3} = (2^3)^{1/3} = 2$

$x^3 = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

$a^{1/n} = \sqrt[n]{a}$

i.e. we can convert between rational exponents and roots/radicals

ex $a^{2/n} = \sqrt[n]{a^2} = (\sqrt[n]{a})^2$ $a^{2/n} = (a^2)^{1/n} = (a^{1/n})^2$

Examples: Simplify these expressions.

(a) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

$\sqrt[4]{81} = \sqrt[4]{9^2} = 9^{2/4} = 9^{1/2}$

$= \sqrt{9} = 3$

(b) $\sqrt[3]{\dfrac{1}{-125}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{-125}} = \dfrac{1}{-5}$

(c) $(-27)^{-4/3} = \dfrac{1}{(-27)^{4/3}} = \dfrac{1}{(\sqrt[3]{-27})^4} = \dfrac{1}{(-3)^4}$

$= \dfrac{1}{81}$

$\dfrac{1}{(-27)^{4/3}} = \dfrac{1}{(-27)^1(-27)^{1/3}}$

$= \dfrac{1}{-27(-3)} = \dfrac{1}{81}$

(d) $9^{3/2}$

$= (\sqrt{9})^3 = 3^3 = 27$

8.1 MC#6) $\left(\frac{4}{25}\right)^{-1/3}, \quad \left(\frac{25}{4}\right)^{1/3}, \quad \left(\frac{4}{25}\right)^{-1/4}$

$$\Longleftrightarrow \left(\frac{25}{4}\right)^{1/3} = \left(\frac{25}{4}\right)^{1/3} > \left(\frac{25}{4}\right)^{1/4}$$

B3) $0.8, \ 0.\overline{8}, \ 0.\overline{89}, \ 0.8\overline{89}, \ \sqrt{0.7744} \doteq 0.88$

$$0.8 < 0.\overline{8} < 0.8\overline{89} < 0.\overline{89}$$

$$0.8 < \sqrt{0.7744} < 0.\overline{8} < 0.8\overline{89} < 0.\overline{89}$$

Decimal WkSt

$100 item

5) (a) 70% off orig. price

(b) 50% off orig. price, + 25% off

(a) pay: $30

(b) pay: $0.75(0.5(100))$

$= 0.75(50) = \$37.50$

6) (a) 10% raise, then 10% cut

(b) 10% cut, then 10% raise

(a) $1.1(100000) = \$110{,}000$

$0.9(110{,}000) = \$99{,}000$

(b) $0.9(100000) = \$90000$

$1.1(90{,}000) = \$99{,}000$

Quiz 12

#4) $65(29) = 1885$

① born 1885, $1949 - 1885 = 64$
② born 1914, $1949 - 1914 = 35$
③ born 1943, $1949 - 1943 = 6$

7 (c) x = total catch
expenses: $44{,}500 + 0.43x$

$44500 + 0.43x = x$
$44500 = 0.57x$
$x = \$78{,}070.18$

3) x = orig. price
he pay $0.7(0.8x) = 487.20$

$0.56x = 487.20$
$x = \$870$

ex $1+2+3+4+\cdots+193+194 = 97(195)$

$1+194=195$

$2+193=195$

$3+192=195$

$\vdots$

$97+98=195$

$= \dfrac{194(195)}{2}$

$1+2+\cdots+n = \dfrac{n(n+1)}{2}$

$11+12+13+\cdots+217 = 103.5(228)$

1) $11+217=228$

2) $12+216=228$

$\vdots$

$108+120=228$

$\vdots$

$112+116=228$

$113+115=228$

114

$$2^4(3)(5^7)+2^4(5^6) = 2^4(5^6)[3\cdot 5 + 1]$$

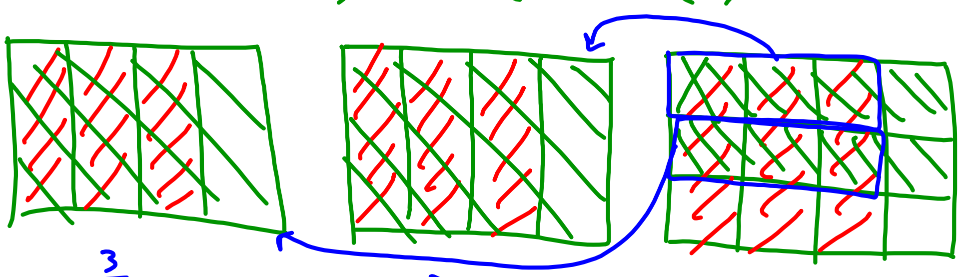
$$= 2^4(5^6)(16)$$

$$= 2^4(5^6)2^4$$

$$= 2^8(5^6) = 2^2(2^6\cdot 5^6) = 2^2(10^6)$$

$$= 4{,}000{,}000$$

$$2\tfrac{2}{3}\cdot\tfrac{3}{4} = \left(2+\tfrac{2}{3}\right)\tfrac{3}{4} = 2\left(\tfrac{3}{4}\right) + \tfrac{2}{3}\left(\tfrac{3}{4}\right)$$



$$\frac{3}{4} \quad + \quad \frac{3}{4} \quad + \quad \frac{6}{12}$$

$$= \frac{9}{12} + \frac{9}{12} + \frac{6}{12} = \frac{24}{12} = 2$$

ex $1\tfrac{2}{5}\cdot\tfrac{3}{4}$

I have $1\tfrac{2}{5}$ yds of fabric.

My kid is smaller than me & her dress requires $\tfrac{3}{4}$ of the material. How much fabric will I use?

$1\tfrac{2}{5} \div \tfrac{3}{4}$

I have $1\tfrac{2}{5}$ yds of fabric.

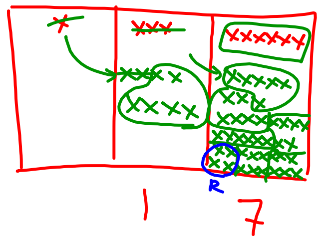
Each pattern needs $\tfrac{3}{4}$ yd.

How many patterns can I make?

Model

$136_8 \div 6_8$

$= 17_8 R 4_8$



Convert 2001_5 to base 10.

$2001_5 = 1(1)+0(5)+0(25)+2(125) = 1+250 = 251$

Convert 2001 to base 5.

625 125 25 5 1

$$\begin{array}{r} 625 \\ \times 3 \\ \hline 1875 \end{array} \qquad \begin{array}{r} 2001 \\ -1875 \\ \hline 126 \end{array}$$

$2001 = 3(625)+1(125)+0(25)+0(5)+1(1)$

$= 31001_5$

ex find GCF + LCM for 135, 21, 1680

3	135	21	1680
5	45	7	560
7	9	7	112
9	9	1	16
16	1	1	16
	1	1	1

GCF = 3

LCM = $3 \cdot 5 \cdot 7 \cdot 9 \cdot 16$

ex convert $1.0\overline{123}$ to a fraction.

$1.0123123123....$

$$\begin{array}{rl} 10000n = & 10123.\overline{123} \\ -\ 10n = & 10.\overline{123} \\ \hline 9990n = & 10,113 \end{array}$$

$n = \frac{10113}{9990} = \frac{3371}{3330} = 1\frac{41}{3330}$