

Math 1210 Midterm Review

(Sections 1.4, 1.5, 1.6, 2.1, 2.2, 2.3)

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uID: _____ Special Number: _____

Instructions: Please show all of your work. All answers should be completely simplified, unless otherwise stated. No calculators or electronics of any kind are allowed.

1. Calculate the following limits. If they are infinite or do not exist, state this.

(a) $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 2x + 4}$

Answer: _____

(b) $\lim_{\theta \rightarrow 0} \frac{\cot(\pi\theta) \cos \theta}{2 \sec \theta}$

Answer: _____

(c) $\lim_{t \rightarrow 0} \frac{\sin^2(3t)}{4t}$

Answer: _____

$$(d) \lim_{x \rightarrow \infty} \sqrt{\frac{x^2+x+3}{(x-1)(x+2)}}$$

Answer: _____

$$(e) \lim_{x \rightarrow \infty} \sqrt[3]{\frac{\pi x^3+7x}{\sqrt{2x^3+3x^2}}}$$

Answer: _____

$$(f) \lim_{x \rightarrow -\infty} \frac{3\sqrt{-x^3}+4x}{\sqrt{-8x^3}}$$

Answer: _____

$$(g) \lim_{x \rightarrow 5^-} \frac{\sin|x-5|}{x-5}$$

Answer: _____

$$(h) \lim_{x \rightarrow 5^+} \frac{\sin |x-5|}{\tan(x-5)}$$

Answer: _____

$$(i) \lim_{x \rightarrow \infty} x^{-1/2} \sin x$$

Answer: _____

$$(j) \lim_{x \rightarrow -\infty} \sin \left(x + \frac{1}{x} \right)$$

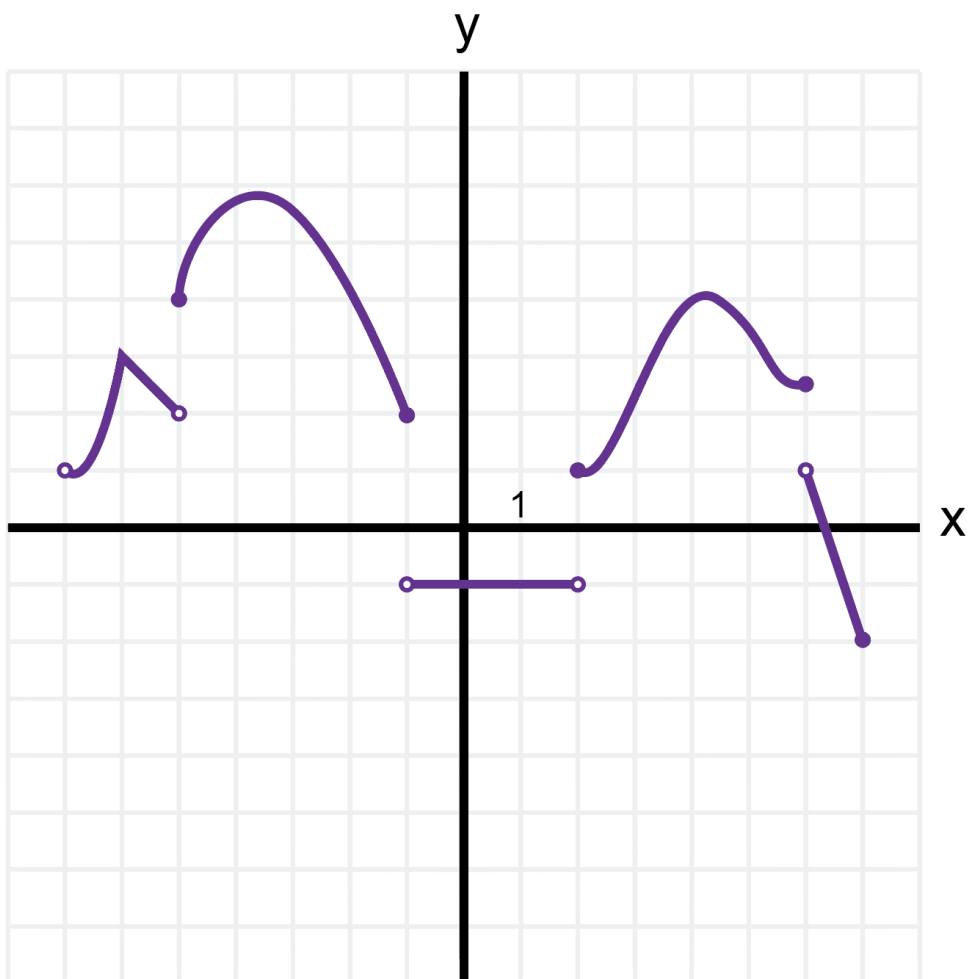
Answer: _____

$$(k) \lim_{x \rightarrow -\infty} \left[\sin \left(x + \frac{1}{x} \right) - \sin x \right]$$

Answer: _____

2. Continuity (Graph-Based)

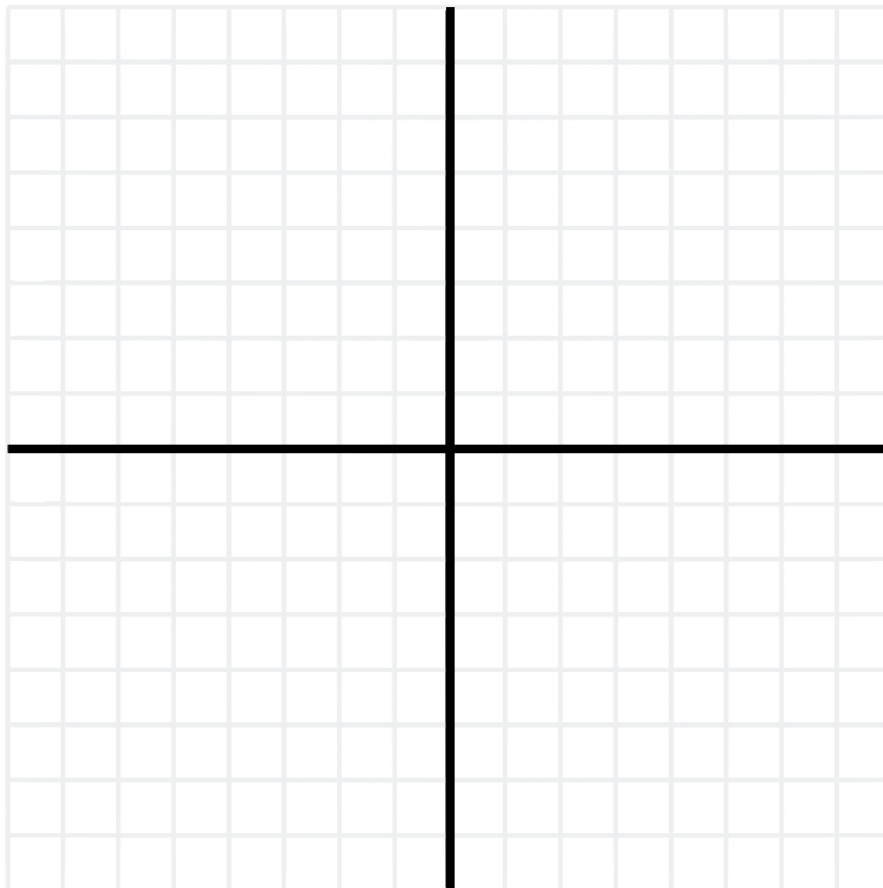
- (a) From the graph of h given below, indicate the intervals on which h is continuous. Indicate left or right continuity when possible.



(b) Sketch the graph of a function $g(x)$ that satisfies the following conditions:

- Its domain is $[-2,2]$.
- It satisfies $f(-2) = f(-1) = f(1) = f(2) = 1$.
- It is discontinuous at -1 and 1 .
- It is right continuous at -1 and left continuous at 1 .

You need not provide an equation for $g(x)$.



3. Constructing Examples

- (a) Give an example of a function such that $f(x)$ is not continuous everywhere, but $|f(x)|$ is.

Answer: _____

- (b) Give an example of a function with discontinuities at infinitely many points.

Answer: _____

- (c) Give an example of a real-valued function that is not continuous on the left half plane.

Answer: _____

4. Find the values of a and b such that the function

$$f(x) = \begin{cases} x + 1 & x < 1 \\ ax + b & 1 \leq x < 2 \\ 3x & x \geq 2 \end{cases}$$

is continuous everywhere.

Answer: _____

5. **Tangent Lines**

Using the definition of the derivative, not shortcuts, find the slopes of the tangent lines to the curve at the point $x = -2$:

(a) $y = x^2 - 3$

Answer: _____

(b) $y = -x^3 + x$

Answer: _____

(c) $y = 2x^3 - x^2 - 6$

Answer: _____

6. **Falling Bodies**

A falling body will fall approximately $10t^2$ meters in t seconds under the influence of gravity.

- (a) How far will an object fall between $t = 1$ and $t = 3$ (assuming it doesn't hit the ground)?

Answer: _____

- (b) What is its average velocity on the interval $0 \leq t \leq 3$?

Answer: _____

- (c) What is its average velocity on the interval $1 \leq t \leq 3$?

Answer: _____

- (d) Find its instantaneous velocity at $t = 3$.

Answer: _____

7. Use “shortcuts” to find $f'(x)$ for the given functions.

(a) $f(x) = \pi x^4 + 2x^2 - 5x + 100$

Answer(a): _____

(b) $f(x) = \frac{3x^2 - x + 10}{2x + 1}$

Answer(b): _____

(c) $f(x) = \frac{2\pi}{x^3} - x^{-4} + \frac{9}{x^7}$

Answer(c): _____

(d) $f(x) = (5x^3 + 1)(x^4 - 2x^2 - \frac{1}{2}x)$

Answer(d): _____

8. (a) Find the equation of the tangent line to $y = x^3 - x + 2$ at $x = -1$.

Answer: _____

- (b) Find all points on the graph of $y = \frac{1}{3}x^3 - 16x + 2$ where the tangent line is horizontal.

Answer: _____