Math 5700 Homework
Logarithms/Exponential Functions and Miscellaneous (Fun) Questions

Rewrite these expressions using log properties.

(1) $\log\left(\frac{p^2 q^3}{r}\right)$

(2) $5\ln|x-2|-\ln|x+2|-3\ln|x|$

(3) $4(\ln x+\ln(x+5))-2\ln(x-5)$

Solve these equations.

(4) $\log(5x)+\log(x-1)=2$

(5) $14e^{3x+2}-50=510$

(6) $e^{2x}-6e^{x}=-8$

(7) $4\ln(3x)=15$

(8) $\log(x-1)=\log(x-2)-\log(x+2)$

(9) $\log(x+2)-\log x=\log(x+5)$

Prove these statements are false.

(10) $\ln(x+y)=\ln x+\ln y$

(11) $\ln(x+y)=\ln(x-y)$

(12) $\ln\left(\frac{x}{y}\right)=\frac{\ln x}{\ln y}$

(13) You are depositing $1000 in a savings account. Which of the following will produce the largest balance in ten years?

(a) 6% annual interest rate, compounded annually
(b) 5.85% annual interest rate, compounded continuously
(c) 5.9% annual interest rate, compounded quarterly

(14) The half-life of radioactive actinium is 22 years. What percent of a present amount of radioactive actinium will remain after 19 years?

(15) What is the greatest divisor of 19! and (19! + 17)?

(16) If you lose 20% on an investment during the first year and gain 25% the following year, what is your net gain over the two years?

(17) How many divisors does the number 2007 have?

(18) A set of encyclopedias (one for each letter) is placed on a bookshelf in alphabetical order from left to right. Each encyclopedia is two inches thick including the front and back covers. Each cover (front or back) is ¼ inch thick. A bookworm eats straight through the encyclopedias, beginning inside the front cover of volume A and ending after eating through the back cover of volume Z. How many inches of book did the bookworm eat?

(19) As $x\to\infty$, the function $\left(\frac{x-3}{x+2}\right)^x$ approaches what?

(20) If this multiplication problem works in base b, what is b?

$$(15_b)(15_b)=321_b$$

(21) If $f(x)=3x^2-x+4$, $f(g(x))=3x^4+18x^3+50x^2+69x+48$, then what is one of the sums of all the coefficients of g(x)?

(a) 8 (b) 1 (c) 3 (d) 7 (e) 0

(22) An equilateral triangle is inscribed in a circle. What is the ratio of the area of the triangle to the area of the circle?

(23) Let $y_1=f(x)=\frac{x+1}{x-1}$; $y_2=f(y_1)$; $y_3=f(y_2)$; ... $y_n=f(y_{n-1})$, for n = 1, 2, 3, 4, ...

Find y_{100} .

(24) In a survey of 115 people, only 20 liked all 3 candies: licorice, chocolate and candy corn. Twenty-four did not like any of the candy, 15 liked only chocolate, 41 disliked chocolate but liked at least one of the other two kinds of candy. If 27 liked exactly 2 of the 3 candies, 11 liked only licorice and 59 like candy corn, how many liked chocolate and licorice, but not candy corn?

(25) Suppose it takes h minutes to fill a bath tub using the hot water faucet and c minutes to fill the same tub using the cold water faucet. Starting with an empty tub, the hot water faucet is turned on and then after 1 minute, the cold water faucet is also turned on. How long will it take to fill the tub?

(26) Suppose a bag contains the six letters of the word "booboo." If you take one letter out of the bag at a time and line them up from left to right, what is the probability that you will spell the word "booboo?"

(27) Solve the cryptarithm, where each letter represents a digit and no digit is represented by two different letters.

$$\begin{array}{r} F\ E\ L\ T \\ +M\ I\ C\ E \\ \hline M\ I\ L\ E\ S \end{array}$$

(28) What is the value of $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+...}}}$?

(29) It is Thanksgiving and the king decides to let out a few prisoners. He sends out his top man and tells him to unlock every cell beginning with the first cell. Deciding this is too much, he immediately sends out his number two man and says, "lock every second cell, beginning with cell #2." He thinks on it again and sends his number three man and says "Change the position of the lock on every third cell. If it is locked, unlock it and if it is unlocked, lock it." He sends his number four man and tells him to change the position of the lock on every fourth cell...then the fifth, sixth and so on. He continues in this indecisive manner all night long. If his men act on these instructions in the order he gave them, who will eventually get out of jail?