

Hint for pg 132 # 6a

$$(a) b_p = \left(1 + \frac{1}{p}\right)^p = 1 + \binom{p}{1} \frac{1}{p} + \binom{p}{2} \left(\frac{1}{p}\right)^2 + \binom{p}{3} \left(\frac{1}{p}\right)^3$$

$$+ \dots + \binom{p}{p-1} \left(\frac{1}{p}\right)^{p-1} + \binom{p}{p} \left(\frac{1}{p}\right)^p$$

$$= 1 + 1 + \frac{p(p-1)}{2! p^2} + \frac{p(p-1)(p-2)}{3! p^3} + \frac{p(p-1)(p-2)(p-3)}{4! p^4}$$

$$+ \dots + \frac{p(p-1)(p-2)\dots(2)}{(p-1)! p^{p-1}} + \frac{p(p-1)(p-2)\dots(1)}{p! (p^p)^{p-1}}$$

$$= 1 + 1 + \frac{1}{2!} \left(\frac{p-1}{p}\right) + \frac{1}{3!} \left(\frac{p-1}{p}\right) \left(\frac{p-2}{p}\right) + \frac{1}{4!} \left(\frac{p-1}{p}\right) \left(\frac{p-2}{p}\right) \left(\frac{p-3}{p}\right)$$

$$+ \dots + \frac{1}{(p-1)!} \left(\frac{p-1}{p}\right) \left(\frac{p-2}{p}\right) \left(\frac{p-3}{p}\right) \dots \left(\frac{2}{p}\right)$$

$$+ \frac{1}{p!} \left(\frac{p-1}{p}\right) \left(\frac{p-2}{p}\right) \dots \left(\frac{1}{p}\right)$$

notice: $\frac{2}{p} = \frac{p-(p-2)}{p}$ and $\frac{1}{p} = \frac{p-(p-1)}{p}$

$$\Rightarrow b_p = 1 + 1 + \frac{1}{2!} \left(1 - \frac{1}{p}\right) + \frac{1}{3!} \left(1 - \frac{1}{p}\right) \left(1 - \frac{2}{p}\right) + \frac{1}{4!} \left(1 - \frac{1}{p}\right) \left(1 - \frac{2}{p}\right) \left(1 - \frac{3}{p}\right)$$

$$+ \dots + \frac{1}{(p-1)!} \left(1 - \frac{1}{p}\right) \left(1 - \frac{2}{p}\right) \left(1 - \frac{3}{p}\right) \dots \left(1 - \frac{p-2}{p}\right)$$

$$+ \frac{1}{p!} \left(1 - \frac{1}{p}\right) \left(1 - \frac{2}{p}\right) \left(1 - \frac{3}{p}\right) \dots \left(1 - \frac{p-1}{p}\right)$$

With very similar work, and noticing that

$$\frac{p}{p+1} = 1 - \frac{1}{p+1}, \quad \frac{p-1}{p+1} = 1 - \frac{2}{p+1}, \dots$$

Then you get

$$b_{p+1} = 1 + 1 + \frac{1}{2!} \left(1 - \frac{1}{p+1}\right) + \frac{1}{3!} \left(1 - \frac{1}{p+1}\right) \left(1 - \frac{2}{p+1}\right)$$

$$+ \frac{1}{4!} \left(1 - \frac{1}{p+1}\right) \left(1 - \frac{2}{p+1}\right) \left(1 - \frac{3}{p+1}\right)$$

$$+ \dots + \frac{1}{p!} \left(1 - \frac{1}{p+1}\right) \left(1 - \frac{2}{p+1}\right) \dots \left(1 - \frac{p-1}{p+1}\right)$$

$$+ \frac{1}{(p+1)!} \left(1 - \frac{1}{p+1}\right) \left(1 - \frac{2}{p+1}\right) \dots \left(1 - \frac{p}{p+1}\right)$$

To compare b_p to b_{p+1} , compare "like" terms, meaning compare the $\frac{1}{2!}$ term in b_p to the $\frac{1}{2!}$ term in b_{p+1} , the $\frac{1}{3!}$ terms together, etc.

and
note:
($p > 0$)

$$p+1 > p \Leftrightarrow \frac{1}{p+1} < \frac{1}{p} \Leftrightarrow 1 - \frac{1}{p+1} > 1 - \frac{1}{p}$$

$$\text{and } 1 - \frac{2}{p+1} > 1 - \frac{2}{p}$$

$$\text{and } 1 - \frac{3}{p+1} > 1 - \frac{3}{p}$$

etc.