Math5700 Trigonometry Questions

1. Verify these identities.

(a) $\dfrac{1}{\sec x+1}-\dfrac{1}{\sec x-1}=-2\cot^2 x$

(b) $\dfrac{\cos(3x)}{\cos x}=1-4\sin^2 x$

(c) $1+\sin^2(\theta)=2-\cos^2(\theta)$

(d) $\dfrac{1+\tan(\theta)}{1-\tan(\theta)}=\dfrac{\sec^2(\theta)+2\tan(\theta)}{2-\sec^2(\theta)}$

2. Determine the area of the triangle ABC with given information.
$$A=38^\circ, \qquad C=100^\circ, \qquad b=15\,\text{meters}$$

3. Find the **exact** value of these expressions (without a calculator!).

(a) $csc\left(\arcsin\left(\dfrac{3}{7}\right)\right)$

(b) $\tan\left(\dfrac{5\pi}{3}\right)$

(c) $\arcsin\left(\dfrac{-\sqrt{3}}{2}\right)$

(d) $\arctan(\tan(135^\circ))$

(e) $\cos\left(\arccos\left(\dfrac{3}{4}\right)\right)$

4. Solve the equations. Give exact answers, not calculator approximations.

(a) $\cot\beta=\cot^2\beta$, $\beta\in[0,2\pi)$

(b) $\cos^2(\theta)-\dfrac{1}{2}=0$, $\theta\in[0,2\pi)$

(c) $\sin(5\theta)=\dfrac{1}{2}$, $\theta\in\mathbb{R}$

5. Use the given information to solve the triangle (give angles in degrees and round to nearest whole numbers).
$$B=102^\circ, \qquad a=4, \qquad c=5$$

6. Find the exact value of the following trigonometric functions given that

$$\cos u = \frac{-2}{5} \quad , \quad \sin v = \frac{-1}{3} \quad , \text{ and both } u \text{ and } v \text{ are in Quadrant 3.}$$

(a) $\cos(u-v)$

(b) $\tan(2v)$

(c) $\cos^2\left(\frac{u}{2}\right)$

7. For $z = -2 + 3i$, answer the following questions.

(a) Find the trigonometric form of z , i.e. in form $z = r(\cos\theta + i\sin\theta)$. (Give θ in degrees and round to the nearest degree. Give r in <u>exact</u> form.)

(b) Find z^4 and write answer in standard form, $a+bi$.

8. Find the cube roots of $-1+\sqrt{3}i$. Write your answer in exact radians.

9. The angle of elevation from the top of an observation tower to a plane is 10 degrees. The plane is two miles high and the height of the observation tower is 150 feet. What is the horizontal distance from the observation tower to the plane?

10. Use half-angle identities to simplify the expression $\dfrac{\sin(4\theta)}{1+\cos(4\theta)}$.

11. Explain why the equation $\cos^2 x + \cos x - 6 = 0$ has no solutions.

12. Write up the method for solving the ambiguous SSA triangle case using (a) Law of Sines and (separately) (b) Law of Cosines (you can use references).
(c) Then use your methods to solve one SSA specific example.