7.1 "Basic" Integration

Ex 1: Compute the following integrals.

(a) $\int \frac{3x}{\sqrt{1-x^4}}\,dx$

(b) $\int \frac{5\tan x}{\sec^2 x - 6}\,dx$

7.1 (continued)

Ex 2: Compute the following integrals.

(a) $\int_0^{3/4} \frac{\sin\sqrt{1-x}}{\sqrt{1-x}}\,dx$

(b) $\int_0^{\frac{\pi}{6}} 2^{\cos x} \sin x\,dx$

7.2 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Ex 1: Compute the following integrals.

(a) $\int_{1}^{5} \sqrt{2x}\ln(x^3)\,dx$

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x\sec^2 x\,dx$

7.2 (continued)

Ex 2: Compute the following integrals.

(a) $\int x \arctan x \, dx$	(b) $\int_0^1 x(x-1)^{50} \, dx$
(c) $\int (x+5)e^{3x-4} \, dx$	(d) $\int \cos(\ln x) \, dx$

7.3 Trigonometric Integrals

Ex 1: Evaluate the integral. $\int \cos^5 x\,dx$	A. Form $\int \sin^n x\,dx$ or $\int \cos^n x\,dx$: If n is odd, use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$. If n is even, use the half-angle identities $\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$.
	B. Form $\int \sin^m x \ \cos^n x\,dx$: If m or n is odd, use Pythagorean identity. If both m and n are even, use half-angle identities.
	C. Form $\int \sin(mx)\ \cos(nx)\,dx$ or $\int \sin(mx)\ \sin(nx)\,dx$: or $\int \cos(mx)\ \cos(nx)\,dx$ Use product identities. $\sin(mx)\cos(nx) = \frac{1}{2}(\sin((m+n)x) + \sin((m-n)x))$ $\sin(mx)\sin(nx) = \frac{-1}{2}(\cos((m+n)x) - \cos((m-n)x))$ $\cos(mx)\cos(nx) = \frac{1}{2}(\cos((m+n)x) + \cos((m-n)x))$

Ex 2: Evaluate the integral.

$\int \cos(2x)\cos(8x)\,dx$

7.3 (continued)

Ex 3: Evaluate the integrals.

(a) $\int \sin^3 x \cos^{10} x\, dx$	(b) $\int \sin(5x)\cos(4x)\, dx$
(c) $\int \sin^2 x \cos^2 x\, dx$	(d) $\int \tan^3 x \sec^{-1/2} x\, dx$

7.4 Rationalizing Substitutions

Ex 1: Evaluate the integral. $\int (5+x)(1-x)^{2/3}\,dx$	A. Form $\sqrt[n]{ax+b}$: Try u-sub with $u=\sqrt[n]{ax+b}$. Important note: This is the root of a LINEAR polynomial, i.e. power on x is 1. B. Form $\sqrt{\text{quadratic polynomial}}$: (a) For $\sqrt{a^2-x^2}$ let $x=a\sin\theta$, $\theta \in[\frac{-\pi}{2},\frac{\pi}{2}]$ (b) For $\sqrt{a^2+x^2}$ let $x=a\tan\theta$, $\theta \in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ (c) For $\sqrt{x^2-a^2}$ let $x=a\sec\theta$, $\theta \in[0,\pi]$, $\theta\neq\frac{\pi}{2}$ C. Extra note of useful integrals to have on your note card: $\int \csc x\,dx=\ln\lvert\csc x-\cot x\rvert+C$ $\int \sec x\,dx=\ln\lvert\sec x+\tan x\rvert+C$

7.4 (continued)

Ex 2: Compute the following integrals.

(a) $\int \frac{\sqrt{9-x^2}}{2x}\,dx$	(b) $\int \frac{x}{\sqrt{2x-3}}\,dx$

7.4 (continued)

Ex 3: Compute these integrals.

(a) $\int \frac{dx}{\sqrt{16+6x-x^2}}$	(b) $\int \frac{2x+5}{x^2+2x+2}\,dx$

7.5 Integration with Partial Fraction Decomposition (PFD)

Note: Only use PFD when (1) the integrand is a rational expression (i.e. both the numerator and denominator are polynomials which means only whole powers of x) AND (2) the denominator factors. And, if the numerator is the same or higher degree than the denominator, you MUST do long division first!!!!

Ex 1: Compute the following integrals.

(a) $\int \frac{1}{(x-1)^2(x+4)^2}\,dx$

Ex 1: (b) $\int \frac{2x^2-3x-36}{(2x-1)(x^2+9)}dx$

Ex 2: Compute the following integrals.

(a) $\int \frac{x^6+4x^3+4}{x^3-4x^2}\,dx$

Ex 2: (b) $\int \frac{x^3}{x^2+x-2}\,dx$

7.6 Integration Strategies (using Integral Tables from the Book)

Ex 1: Use substitution and the integral tables from the back of the book to compute these integrals.

(a) $\int \frac{\sin x}{\cos x \sqrt{5-4\cos x}} dx$	Which formula #? ______________
(b) $\int \frac{2x}{5x^4-10} dx$	Which formula #? ______________

Ex 2: Use substitution and the integral tables from the back of the book to compute these integrals.

(a) $\int \frac{dx}{\sqrt{25-e^{10x}}}$ Which formula #? _____________

(b) $\int 3\sqrt{49-e^{10x}}\,dx$ Which formula #? _____________