Math1220 Extra Final Review

Kelly MacArthur

1. Evaluate these integrals.

(a) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

(b) $\int_0^2 \frac{1}{(x-1)^2}\,dx$

(c) $\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx$

(d) $\int x\sin x\,dx$

2. Find the area of the region enclosed by $r=1+\sin\theta$.

3. Solve $\frac{dy}{dt}+4y=8$ given $y=4 \text{ when } t=0$.

4. What is the sum of $1+\frac{\pi}{4}+\frac{\pi^2}{16}+\frac{\pi^3}{64}+\ldots$?

5. Find the area of the region outside $r=2+2\cos\theta$ and inside $r=2$.

6. (a) Find the first four non-zero terms of $f(x)=e^{x^4}$ in its Maclaurin series.

(b) Use that answer (from (a)) to estimate $\int_0^{\frac{1}{2}} e^{x^4}\,dx$.

7. Evaluate these integrals.

(a) $\int \frac{x^2}{x^3+1}\,dx$

(b) $\int y e^{-2y}\,dy$

(c) $\int_0^{\pi} (\sin\theta+\cos\theta)\,d\theta$

(d) $\int x^2 e^{x^3}\,dx$

(e) $\int \frac{x+1}{x(x-1)(x^2+1)}\,dx$

8. For a new radioactive substance, it is found that after 10 years, 5% of the substance has decayed. Find the half life of the substance.

9. Find the convergence set for the following power series.

(a) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n+1}$

10. Does $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 3^{n+1}}{e^{2n}}$ converge absolutely, conditionally or diverge?

11. Find a power series for $F(x)=\int x e^{x^3}\,dx$ and state its radius of convergence.

12. Find $\frac{dy}{dx}$ for each function. (Do not bother simplifying your answers! And make sure your answers are given as y = some function of x.)

(a) $y=\frac{\ln(3x+1)}{\cos(\sqrt{x})}$

(b) $y=(x^2+2)^{1+x}$

13. Solve this differential equation for y. $\frac{dy}{dx}=\frac{x^3-4y}{x}$

14. Evaluate these integrals.

(a) $\int \frac{\sqrt{4x^2+9}}{x^4}\,dx$

(b) $\int_2^4 \frac{dx}{\sqrt{x-2}}$

(c) $\int_0^{\pi/3} x\cos(3x)\,dx$

(d) $\int \frac{2x+3}{x^3+x}\,dx$

(e) $\int \cos^3 x\,dx$

(f) $\int_{-\infty}^{0} 8x^2 e^{-x^3}\,dx$

(g) $\int_0^1 \frac{4y}{\sqrt{y^2+6}}\,dy$

(h) $\int e^x \sin(e^x)\,dx$

(i) $\int \frac{x+9}{x^3+9x}\,dx$

(j) $\int \frac{\cos x(\sin x+\cos x)}{\sin x}\,dx$

(k) $\int_0^4 \frac{x}{\sqrt{9+x^2}}\,dx$

(l) $\int_3^7 \frac{2x}{\sqrt{x-3}}\,dx$

(m) $\int x^2 \ln x\,dx$

(n) $\int_{\frac{1}{2}}^2 \frac{dx}{x\sqrt[3]{\ln x}}$

15. Find the limit, if it exists.

(a) $\lim_{x\to 0} \frac{\sin(5x)}{\tan(4x)}$

(b) $\lim_{x\to\infty} (2x)^{\frac{1}{3x}}$

(c) $\lim_{x\to 0} (1+\sin x)^{\frac{2}{x}}$

(d) $\lim_{x\to 0} \frac{\sin x - \tan x}{x^2 \sin x}$

16. For the sequence given by $a_n = \frac{5n}{\sqrt{5n^2-3}}$, write out the first three terms of the sequence. And, determine if the sequence converges or diverges. If it converges, find its limit.

17. Determine if each series is absolutely convergent, conditionally convergent or divergent. (Show your work to convince me of your result and state which tests you're using to get to your answer.)

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{n^3+5}$

(b) $\sum_{n=1}^{\infty} \frac{n-2}{4n+1}$

(c) $\sum_{n=1}^{\infty} \frac{n^4(-3)^n}{(n+2)!}$

(d) $\sum_{n=1}^{\infty} \frac{(-3)^n n^2}{(2n)!}$

(e) $\sum_{n=1}^{\infty} \frac{2n+7}{\sqrt[4]{4n^4+5n+1}}$

(f) $\sum_{n=1}^{\infty} \frac{3n^3+2n}{1+n^3}$

18. Find a power series that represents $f(x)=\frac{1}{1-2\mathrm{x}}+e^{-3\mathrm{x}}$. (Write out the terms through x^3.) State its radius of convergence.

19. For $f(x)=\frac{1}{3+x}$

(a) Find the Taylor polynomial of order 3 centered about a = 2.
(b) Approximate f(2.3) using the Taylor polynomial in part (a).
(c) Find a bound for the error in your approximation.

20. For Cartesian coordinates $(-3\sqrt{3}\ ,\ 3)$, find three different ways to represent this point in polar coordinates.

21. Find the slope of the tangent line to the graph $r=2+3\sin\theta$ at the point where $\theta=\frac{\pi}{4}$.

22. For the rectangular (Cartesian) equation $x^2+(y-2)^2=4$

(a) Find the equivalent polar equation for this curve.
(b) **Using the polar equation in part (a)**, find the area of the region bounded by the curve. (Hint: You may want to graph the function to ensure you get the correct integration bounds.)

23. Find the derivative. $D_x\left(x^{1+x}\right)$

24. For $f(x)=x^5+2\mathrm{x}^3+4\mathrm{x}$, determine if it's an invertible function. If it is invertible, find $(f^{-1})'(7)$.

25. Find the equation of the tangent line to $f(x)=(1+\sin x)^{\cos x}$ at $x=\frac{\pi}{2}$.

26. Solve the differential equation $\frac{dy}{dx}+2\mathrm{xy}-2\mathrm{x}=0$ if it goes through (0, 3).

27. For the sequence given by $a_n=\frac{1}{\sqrt[3]{n}}+\frac{1}{\sqrt[n]{3}}$, write out the first three terms of the sequence. And, determine if the sequence converges or diverges. If it converges, find its limit.

28. For $f(x)=\frac{2}{x-1}$

(a) Find the Taylor polynomial of order 4 centered about a = 2.
(b) Approximate f(1.5) using the Taylor polynomial in part (a).
(c) Find a bound for the error in your approximation.

29. Convert the point $\left(-1\ ,\ \frac{5\pi}{4}\right)$ from polar coordinates to rectangular coordinates.

30. For the function (in polar coordinates) given by $r^2 - 6r\cos\theta - 4r\sin\theta + 9 = 0$
(a) convert this function to rectangular coordinates, and
(b) describe the shape.

31. Find a power series that represents $f(x) = \frac{3x^2}{4 - x^3}$ and state its radius of convergence.

Answers:

1. (a) $\frac{-2}{1+\sqrt{x}}+C$
(b) diverges
(c) 2
(d) $\sin x - x\cos x + C$

2. $\frac{3\pi}{2}$

3. $y=2+\frac{2}{e^{4t}}$

4. $\frac{4}{4-\pi}$

5. $8-\pi$

6. (a) $1+x^4+\frac{1}{2}x^8+\frac{1}{6}x^{12}+\frac{1}{24}x^{16}+...$
(b) ~0.50636007

7. (a) $\frac{1}{3}\ln|x^3+1|+C$
(b) $\frac{-1}{2}x e^{-2x}-\frac{1}{4}e^{-2x}+C$
(c) 2
(d) $\frac{1}{3}e^{x^3}+C$
(e) $\ln\left|\frac{x-1}{x}\right|-\arctan x+C$

8. ~135 years

9. (a) $x\in\mathbb{R}$
(b) (1, 5)

10. converges absolutely

11. $\sum_{n=0}^{\infty}\frac{x^{3n+2}}{n!(3n+2)}$

12. (a) $\frac{dy}{dx}=\frac{\cos(\sqrt{x})\left(\frac{3}{3x+1}\right)+\frac{\ln(3x+1)\sin(\sqrt{x})}{2\sqrt{x}}}{\cos^2(\sqrt{x})}$
(b) $\frac{dy}{dx}=(x^2+x)^{1+x}\left(\frac{2x(1+x)}{x^2+2}+\ln(x^2+x)\right)$

13. $y=\frac{1}{7}x^3+\frac{c}{x^4}$

14. (a) $\frac{-(4x^2+9)^{\frac{3}{2}}}{27x^3}+C$

(b) $2\sqrt{2}$

(c) $\frac{-2}{9}$

(d) $3\ln|x|-\frac{3}{2}\ln(x^2+1)+2\arctan x+C$

(e) $\sin x-\frac{1}{3}\sin^3 x+C$

(f) diverges

(g) $4(\sqrt{7}-\sqrt{6})$

(h) $-\cos(e^x)+C$

(i) $\ln|x|-\frac{1}{2}\ln(x^2+9)+\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

(j) $\sin(x)+\cos(x)+\ln|csc(x)-\cot(x)|+C$

(k) 2

(l) $\frac{104}{3}$

(m) $\frac{1}{3}x^3(\ln x-\frac{1}{3})+C$

(n) 0

15. (a) $\frac{5}{4}$

(b) 1

(c) e^2

(d) $-\frac{1}{2}$

16. $a_1=\frac{5}{\sqrt{2}}$, $a_2=\frac{10}{\sqrt{17}}$, $a_3=\frac{15}{\sqrt{42}}$, converges to $\sqrt{5}$

17. (a) converges absolutely

(b) diverges

(c) converges absolutely

(d) converges absolutely

(e) diverges

(f) diverges

18. $2-x+\frac{17}{2}x^2+\frac{7}{2}x^3+\frac{155}{8}x^4+...$, radius of convergence is $\frac{1}{2}$

19. (a) $\frac{1}{5}-\frac{1}{25}(x-2)+\frac{1}{125}(x-2)^2-\frac{1}{625}(x-2)^3+...$

(b) 0.1886768

(c) $|R_4(2.3)|\leq 0.000002592$

20. $(6,\frac{5\pi}{6})$, $(-6,\frac{-\pi}{6})$, $(6,\frac{-7\pi}{6})$

21. $\frac{-2-3\sqrt{2}}{2}$

22. (a) $r=4\sin\theta$

(b) 4π

23. $x^{1+x}\left(\frac{1+x}{x}+\ln x\right)$

24. $\frac{1}{15}$

25. $y=(-\ln 2)x+1+\frac{\pi}{2}\ln 2$

26. $y=1+2e^{-x^2}$

27. $a_1=\frac{4}{3}$, $a_2=\frac{3\sqrt[3]{4}+2\sqrt{3}}{6}$, $a_3=\frac{2}{\sqrt[3]{3}}$ $\left(\text{or } \frac{2\sqrt[3]{9}}{3}\right)$; converges to 1

28. (a) $f(x)\approx 2-2(x-2)+2(x-2)^2-2(x-2)^3+2(x-2)^4$

(b) $\frac{31}{8}$

(c) 4

29. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

30. (a) $(x-3)^2+(y-2)^2=4$

(b) it's a circle with radius of 2 and center at (3, 2)

31. $\sum_{n=0}^{\infty}\frac{3x^{3n+2}}{4^{n+1}}$; radius of convergence = $\sqrt[3]{4}$