M1220

6.1 Practice (Natural Logarithm)

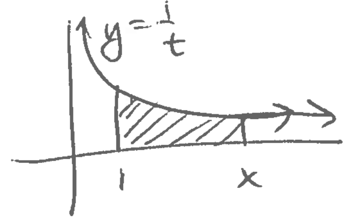
Evaluate.

Ex 1 (a) $D_x\left(\ln\left(3x-\sqrt{x^3+1}\right)\right)$

(b) $\int_0^{\pi/3} \tan x \, dx$

Defn

$\ln x = \int_1^x \frac{1}{t}\,dt \quad , x>0$



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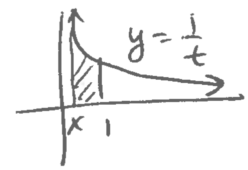


$\Rightarrow D_x(\ln x)=\frac{1}{x} \quad , x>0$

$D_x(\ln|x|)=\frac{1}{x}, \quad x\neq 0$

$$\boxed{\Rightarrow \int \frac{1}{\heartsuit}\,d\heartsuit = \ln|\heartsuit| + C}$$

Properties

$a,b\in\mathbb{R}^+, \quad r\in\mathbb{Q}$

1. $\ln 1 = 0$
2. $\ln(ab)=\ln a+\ln b$
3. $\ln\left(\frac{a}{b}\right)=\ln a-\ln b$
4. $\ln a^r = r\ln a$

6.1

Ex 2

(a) $D_x\left(\ln(\sec^2 x)\right)$

(b) $D_x\left(\dfrac{x^2+4}{9-\ln(3x^2-5)}\right)$

Ex 3

(a) $\displaystyle\int \frac{9y}{5y^2-1}\,dy$

(b) $\displaystyle\int \frac{x^3+x^2}{x+2}\,dx$

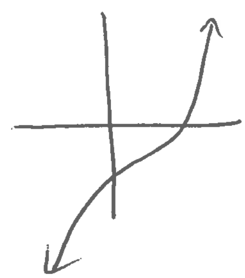
6.1

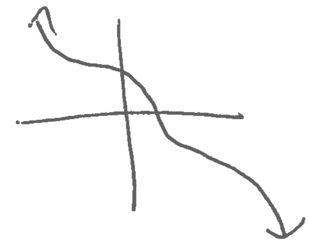
Ex 4 Explain why $\lim_{x\to 0} \left(\ln\left(\frac{\sin x}{x}\right)\right) = 0$.

6.2 Practice (Inverse Fns & Their Derivatives)

Ex 1 (a) Show $f^{-1}(x)$ exists for $f(x) = \int_x^1 \sin^4 t \, dt$.

If f is strictly monotonic on its domain, $f^{-1}(x)$ exists.





$f^{-1}(f(♡)) = ♡$

f differentiable & monotonic on I. If $f'(x) \neq 0$, then $f^{-1}(x)$ is differentiable at $y = f(x)$ and $(f^{-1})'(y) = \frac{1}{f'(x)}$

i.e. $\frac{dx}{dy} = \frac{1}{dy/dx}$.

6.2

Ex1 (b) Find $(f^{-1})'(2)$ for $f(x)=\sqrt{3x-2}$, if possible.

Ex2 Prove $f(x)=-x^9-4x^3+10$ has an inverse.

Ex3 Find $(f^{-1})'(15)$ for $f(x)=-x^9-4x^3+10$.

6.3 Practice (Natural Exponential Function)

$e^{\ln x} = x \quad x > 0$

$\ln(e^x) = x \quad \forall x$

$\int e^{♡} d♡ = e^{♡} + C$

$D_x(e^x) = e^x$

Ex 1 (a) Simplify. $e^{\ln w^3 - 4 \ln u}$

(b) Find $\frac{dy}{dx}$ for $3y = 4e^{xy} - 2\cos x$.

(c) Evaluate $\int_0^1 3x\, e^{6x^2-1}\, dx$

$\boxed{6.3}$

Ex 2 For $f(x) = 2x + e^x$, find min/max pts, inflection pts, any asymptotes, & sketch graph.

Ex 3 (a) $\int \frac{e^x}{e^x - 3}\, dx$

(b) $D_x\left(e^{x^3 \cos x}\right)$

6.4 Practice (General Exponential / Logarithmic Fns)

$[\ln(5x-1)]^{3x^2}$

Ex1 (a) $D_x\left[\left(\cancel{\log_7(x^5-1)}\right)\sin x\right]$

$D_x(a^x)=a^x\ln a$

$\int a^x\,dx=\left(\frac{1}{\ln a}\right)a^x+C$

$(a\neq 1)$

$a>0,\ a\neq 1$

$y=\log_a x \iff x=a^y$

$D_x(\log_a x)=\frac{1}{x\ln a}$

Bonus w/ this news:

$\int x^a\,dx=\frac{x^{a+1}}{a+1}+C$

$\forall\ a\neq -1$

(b) $\int 8^{2x-1}\,dx$

(c) $\int (2x-1)^8\,dx$

6.4

Ex 2 (a) $D_x\left(\sin^3 x + 3^{\sin x}\right)$

(b) $D_x\left((x^2-1)^{\tan x}\right)$

Ex 3 (a) $\int_1^{25} \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b) $\int\left(9^{2x} - 3^{4x}\right)dx$

6.5 Practice (Exponential Growth & Decay)

$\frac{dy}{dt} = ky \Rightarrow y = y_0 e^{kt}$

☆ $\lim_{h \to 0} (1+h)^{1/h} = e$

Ex 1 A population is growing at a rate proportional to its size. After 10 years, its population was 250,000. After 20 years, the population was 800,000. What was the original population?

Ex 2 (a) $\lim_{x \to 0} (1+x)^{1000000}$

(b) $\lim_{x \to 0} 1^{\frac{1}{x}}$

$\boxed{6.5}$

Ex2 (a) $\lim_{x \to 0} (1+5x)^{1/x}$

(b) $\lim_{x \to 0^+} (1+\varepsilon)^{1/x}$, $\varepsilon > 0$

(c) $\lim_{x \to 0^-} (1+\varepsilon)^{1/x}$, $\varepsilon > 0$

6.8 Practice (Inverse Trigonometric Fns & Their Derivatives)

Ex 1 Evaluate.

(a) $\tan\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$

Trig Review

(A)
$x = \sin^{-1} y \iff y = \sin x$, $x \in [-\pi/2, \pi/2]$

$x = \cos^{-1} y \iff y = \cos x$, $x \in [0, \pi]$

$x = \tan^{-1} y \iff y = \tan x$, $x \in (-\pi/2, \pi/2)$

$x = \sec^{-1} y \iff y = \sec x$, $x \in [0, \pi], x \neq \frac{\pi}{2}$

(B)
$\sec^{-1} y = \cos^{-1}\left(\frac{1}{y}\right)$

$\cot^{-1} y = \tan^{-1}\left(\frac{1}{y}\right)$

$\csc^{-1} y = \sin^{-1}\left(\frac{1}{y}\right)$

(C)
$\sin(\cos^{-1} x) = \sqrt{1-x^2}$

$\cos(\sin^{-1} x) = \sqrt{1-x^2}$

$\sec(\tan^{-1} x) = \sqrt{1+x^2}$

$$\tan(\sec^{-1} x) = \begin{cases} \sqrt{x^2-1}, & x \geq 1 \\ -\sqrt{x^2-1}, & x \leq -1 \end{cases}$$

New

$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\quad x \in (-1,1)$

$D_x(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ $\quad x \in (-1,1)$

$D_x(\tan^{-1} x) = \frac{1}{1+x^2}$

$D_x(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\quad |x| > 1$

CALCULUS Review

$D_x(\sin x) = \cos x$

$D_x(\cos x) = -\sin x$

$D_x(\tan x) = \sec^2 x$

$D_x(\sec x) = \sec x \tan x$

$D_x(\csc x) = -\csc x \cot x$

$D_x(\cot x) = -\csc^2 x$

$\boxed{6.8}$

Ex1 (b) $\lim_{x \to 1} \sin^{-1} x$.

(c) $D_x \left(x \sin^{-1}(x^2+1) \right)$

(d) $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

$\boxed{6.8}$

<u>Ex 2</u> $D_x\left(e^x \cos^{-1}(x^3)\right)$

<u>Ex 3</u> (a) $\displaystyle\int \frac{x}{\sqrt{12-9x^2}}\,dx$

(b) $\displaystyle\int \frac{1}{x^2+8x+18}\,dx$

6.9 Practice (Hyperbolic fns + Their Inverses)

Ex 1 (a) $D_x(\ln(\operatorname{sech} x + \sinh x))$

(b) $y = \cosh x$, $y = 0$, $x = 0$, $x = 1$ region is revolved about x-axis. Find volume. (hint: $\cosh^2 x = \frac{1}{2}(1 + \cosh(2x))$)

$\cosh^2 x - \sinh^2 x = 1$

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$D_x(\sinh x) = \cosh x$

$D_x(\cosh x) = \sinh x$

$D_x(\tanh x) = \operatorname{sech}^2 x$

$D_x(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$D_x(\coth x) = -\operatorname{csch}^2 x$

$D_x(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$

$\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$, $x \geq 1$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ $x \in (-1, 1)$

$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$, $x \in (0, 1]$

$D_x(\sinh^{-1} x) = \dfrac{1}{\sqrt{x^2+1}}$

$D_x(\cosh^{-1} x) = \dfrac{1}{\sqrt{x^2-1}}$ $(x > 1)$

$D_x(\tanh^{-1} x) = \dfrac{1}{1-x^2}$ $x \in (-1, 1)$

$D_x(\operatorname{sech}^{-1} x) = \dfrac{-1}{x\sqrt{1-x^2}}$ $x \in (0, 1)$

6.9

Ex 2 (a) $D_x\left(\cos^{-1}(\cosh(2x))\right)$

(b) $D_x\left(\frac{\sinh x}{1+\ln x}\right)$

Ex 3 (a) $\int 2x\, \operatorname{sech}^2(x^2-3)\, dx$

(b) Find area bounded by $y=\tanh x$, $y=0$, $x=-8$, $x=8$.

7.1 Practice ("Basic" Integration)

Ex 1 (a) $\displaystyle\int \frac{3x\,dx}{\sqrt{1-x^4}}$

(b) $\displaystyle\int \frac{5\tan x}{\sec^2 x - 6}\,dx$

Ex 2 (a) $\displaystyle\int_0^{3/4} \frac{\sin\sqrt{1-x}}{\sqrt{1-x}}\,dx$

(b) $\displaystyle\int_0^{\pi/6} 2^{\cos x} \sin x\,dx$