

## 7.2 Practice (Integration By Parts)

Ex 1 (a)  $\int_1^5 \sqrt{2x} \ln x^3 dx$

$$\int u dv = uv - \int v du$$

(b)  $\int_{\pi/6}^{\pi/4} x \sec^2 x dx$

(7.2)

Ex 2

(a)  $\int x \arctan x \, dx$

(b)  $\int \cos(\ln x) \, dx$

Ex 3 (a)  $\int_0^1 x (x-1)^{50} \, dx$

(b)  $\int (t+5) e^{3t-4} \, dt$

## 7.3 Practice (Trigonometric Integrals)

Ex 1 (a)  $\int \cos^5 x \, dx$

(b)  $\int \cos(2x) \cos(9x) \, dx$

①  $\int \sin^n x \, dx$  or  $\int \cos^n x \, dx$

Try (a)  $\sin^2 x + \cos^2 x = 1$  if  
n odd

half-angle identities  $\left\{ \begin{array}{l} \text{(b) } \sin^2 x = \frac{1 - \cos(2x)}{2} \\ \text{and/or } \cos^2 x = \frac{1 + \cos(2x)}{2} \end{array} \right.$   
if n even

②  $\int \sin^m x \cos^n x \, dx$

Try (a)  $\sin^2 x + \cos^2 x = 1$   
if m or n odd

(b) half-angle identities if  
both m and n even

③  $\int \sin(mx) \cos(nx) \, dx$

$\int \sin(mx) \sin(nx) \, dx$

$\int \cos(mx) \cos(nx) \, dx$

Try product identities:

$$\sin(mx) \cos(nx) = \frac{1}{2} [\sin(m+n)x + \sin(m-n)x]$$

$$\sin(mx) \sin(nx) =$$

$$-\frac{1}{2} [\cos(m+n)x - \cos(m-n)x]$$

$$\cos(mx) \cos(nx) = \frac{1}{2} [\cos(m+n)x + \cos(m-n)x]$$

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(c)  $\int \tan^3 x \sec^{-1/2} x dx$

④  $\int \tan^n x dx, \int \cot^n x dx$

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Try Pythagorean

Identities:

(a)  $\tan^2 x = \sec^2 x - 1$

(b)  $\cot^2 x + 1 = \csc^2 x$

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⑤  $\int \tan^m x \sec^n x dx$

$\int \cot^m x \csc^n x dx$

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Try

(a) use Pythagorean identities if  $n$  even  
( $m$  any number)

(b) use Pythagorean identities (factoring out one  $\tan x$  or  $\cot x$ ) if  $m$  odd  
( $n$  any number)

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Ex 2 (a)

$$\int \sin(5x) \sin(4x) dx$$

(b)  $\int \cot^5(2t) dt$

Ex 3 (a)

$$\int \sin^2 x \cos^2 x dx$$

(b)  $\int \sin^3 x \cos^{10} x dx$

## 7.4 Practice (Rationalizing Subs)

Ex 1 (a)  $\int (5+x)(1-x)^{2/3} dx$

①  $\sqrt[n]{ax+b}$   
Try  $u = \sqrt[n]{ax+b}$

②  $\sqrt{a^2-x^2} \Rightarrow x = a \sin t$   
(a)  $t \in [-\pi/2, \pi/2]$

(b)  $\sqrt{a^2+x^2} \Rightarrow x = a \tan t$   
 $t \in (-\pi/2, \pi/2)$

(c)  $\sqrt{x^2-a^2} \Rightarrow x = a \sec t$   
 $t \in [0, \pi], t \neq \pi/2$

(b)  $\int \frac{dx}{\sqrt{16+6x-x^2}}$

Extra Note!

$$\int \csc x dx = \ln |\csc x - \cot x| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

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Ex2

$$(a) \int \frac{u \, du}{\sqrt{2u-3}}$$

$$(b) \int \frac{\sqrt{9-x^2}}{2x} \, dx$$

$$\text{Ex3} \int \frac{2x+1}{x^2+2x+2} \, dx$$

# 7.5 Practice (Integration w/ Partial Fraction Decomposition) (PFD)

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Ex 1 (a)  $\int \frac{x^6 + 4x^3 + 4}{x^3 - 4x^2} dx$

(b)  $\int \frac{x^3}{x^2 + x - 2} dx$



7.5

Ex 2  $\int \frac{1}{(x-1)^2(x+4)^2} dx$

Ex 3  $\int \frac{2x^2 - 3x - 36}{(2x-1)(x^2+9)} dx$

## 7.6 Practice (Integration Strategies)

\* you may need integration table from book

$$\text{Ex 1 (a)} \quad \int_3^4 \frac{1}{t - \sqrt{2t}} dt$$

$$\text{Ex 2 (a)} \quad \int \frac{x}{5x^4 - 10} dx$$

$$(b) \quad \int_0^1 x^3 e^{-x} dx$$

$$(b) \quad \int \frac{\sin x}{\cos x \sqrt{5 - 4 \cos x}} dx$$