MATH 1050-90
PRACTICE EXAM 2
(Sections 2.6, 3.1-3.5, 7.1-7.6)

The purpose of the practice exam is to give you an idea of the following:

- *length of exam*
- *difficulty level of problems*

Your actual exam will have different problems. You should review your homework, quizzes, notes etc., not just perfect taking the practice exam. However, you can use the practice exam to gauge how well you know the material. To do this, take it under the same conditions as a normal exam (no notes, no calculator, time yourself). Then score your problems against the solution key.

The following instructions are on the exam:

- Use a PENCIL, erase or cross out errors. If you use a pen, please use white out tape or neatly cross out work you don't want graded.
- **SHOW ALL WORK**. No points will be given for answers without justification.
- NO CALCULATORS, NOTES, PHONES, ETC.
- Answers should be simplified (reduced).
- The value of each question is shown.
- Finish in 80 minutes (one hours + 20 minute grace period). You are responsible for keeping track of the time. The proctor does not say "time is up". For each minute you take beyond 80 min, your score will be reduced by 0.5%.

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This will appear on the exam!

Formulas:

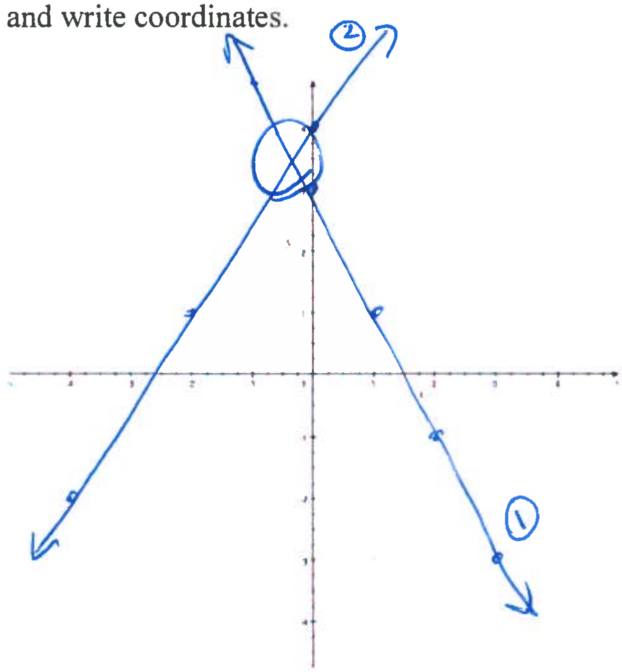
$y = Pe^{rt}$ and $y = P\left(1+\frac{r}{n}\right)^{nt}$

Remember log(a) means log in base 10, and ln(a) means base *e*.

1. Solve this problem by graphing, then by an algebraic method of your choice. (*10 pts)*

$-3x + 2y = 8 \Rightarrow y = \frac{3}{2}x + 4$ ②
$2x + y = 3 \Rightarrow y = -2x + 3$ ①

Graph it: Circle solution and write coordinates.



Solve algebraically:
Write the solution as an ordered pair. pair, (a,b).

① $y = 3 - 2x$

② $-3x + 2(3-2x) = 8$

$-3x + 6 - 4x = 8$

$-7x + 6 = 8$

$-7x = 2$

$x = -2/7$

$\Rightarrow y = 3 - 2\left(\frac{-2}{7}\right) = 3 + \frac{4}{7} = \frac{21}{7} + \frac{4}{7}$

$= \frac{25}{7}$ or $3\frac{4}{7}$

$\boxed{\left(\frac{-2}{7}, \frac{25}{7}\right)}$

2. Find the points of intersection for the following set of simultaneous equations. State the answer as ordered pairs, (a,b). *(8 pts)*

Solve Algebraically:

① $y = x^2 + 5x$
② $y = -x^2 + 3$ } add

$\boxed{(-3, -6) \quad \left(\frac{1}{2}, \frac{11}{4}\right)}$

$2y = 5x + 3$

$\Rightarrow y = \frac{5}{2}x + \frac{3}{2} \Longrightarrow$ ① $2\left(\frac{5}{2}x + \frac{3}{2}\right) = (x^2 + 5x)2$

$5x + 3 = 2x^2 + 10x$

$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$x = 1/2$, or $x = -3$

if $x = \frac{1}{2}$, $y = \frac{5}{2}\left(\frac{1}{2}\right) + \frac{3}{2}$
$= \frac{5}{4} + \frac{6}{4} = \frac{11}{4}$

if $x = -3$, $y = \frac{5}{2}(-3) + \frac{3}{2}$
$= \frac{-12}{2} = -6$

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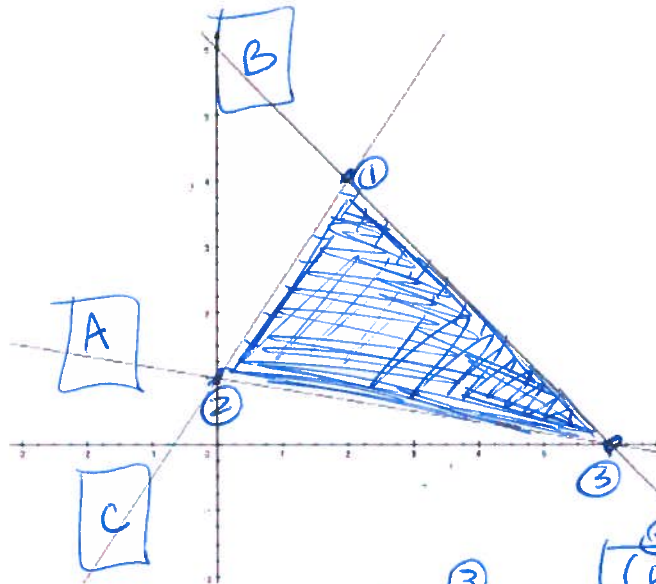
3. Linear Programming: *(10 pts)*
   a. Label the three lines on the graph with A, B, C to show which constraint they represent.
   b. Shade the area represented by the three constraints.

Constraints:

$A: \quad x+6y \geq 6$ $\Rightarrow$ $y \geq -\frac{1}{6}x+1$

$B: \quad 2x+2y \leq 12$ $\Rightarrow$ $y \leq -x+6$

$C: \quad y \leq \frac{3}{2}x+1$



2 A and C

$-\frac{1}{6}x+1 = \frac{3}{2}x+1$

$6(-\frac{1}{6}x+1) = (\frac{3}{2}x+1)6$

$-x+6 = 9x+6$

$-10x = 0$ $\Rightarrow y = \frac{3}{2}(0)+1 = 1$

$x = 0$

2 $\boxed{(0,1)}$

(we also could have just seen this)

3 A and B

$-x+6 = -\frac{1}{6}x+1$

$-\frac{5}{6}x = -5$

$x = 6$

$y = -6+6 = 0$

3 $\boxed{(6,0)}$

   c. List the three vertex points.

1 B and C

$y = -x+6$
$y = \frac{3}{2}x+1$

$-x+6 = \frac{3}{2}x+1$

$-\frac{5}{2}x = -5$

$x = 2 \Rightarrow y = -2+6 = 4$

1 $\boxed{(2,4)}$

   d. Test the objective equation z = y-3x for each of the vertex points and determine where the maximum and minimum occur.

1 $(2,4)$ $\quad z = 4-3(2) = -2$

2 $(0,1)$ $\quad z = 1-3(0) = 1$

3 $(6,0)$ $\quad z = 0-3(6) = -18$

$\boxed{\text{max at } (0,1) \quad \text{min at } (6,0)}$

4. Solve for x, stating exact answer in simplified form. *(6 pts)*
   Show one step in the process.

a. $\log_x 36 = 2$

$x^2 = 36$

$\Rightarrow \boxed{x = 6}$

b. $2^x = \frac{1}{32}$

$2^x = 2^{-5}$

$\boxed{x = -5}$

c. $\log_2 64 = x$

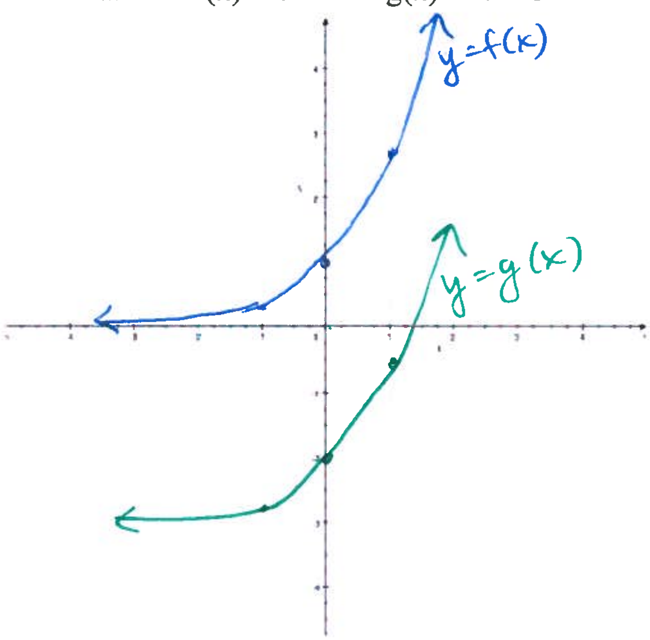
$2^x = 64 = 2^6$

$x = 6$

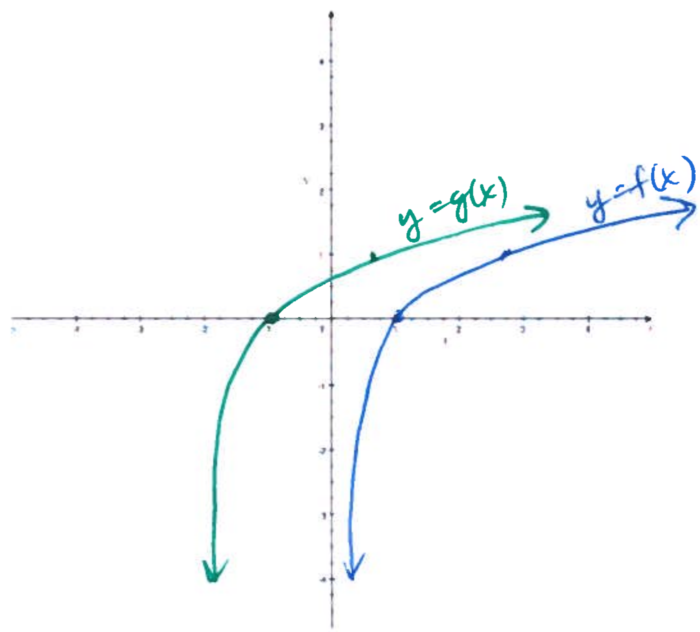
Score on Page ________

5. Sketch both functions on the same graph (in different colors if you have them). Label the graphs and show the asymptotes: *(8 pts)*

a. $f(x) = e^x$ $g(x) = e^x - 3$ (shift down 3)



b. $f(x) = \ln(x)$ $g(x) = \ln(x+2)$ shift left 2



6. A substance is growing continuously. If you start with 15 g and eight weeks later you have 75 g, what is the rate of growth? (State an exact answer.) *(5 pts)*

$y = Pe^{rt}$

$P = 15g$

$t = 8wk,\ y = 75g$

$r = ?$

$75 = 15e^{8r}$

$5 = e^{8r}$

$\ln 5 = 8r$

$\boxed{r = \frac{1}{8}\ln 5}$

7. Simplify the left side of this equation to a single log expression and solve for x: *(5 pts)*

$\log(x-6) - \log(2x+1) = 0$ $x = ?$

$\log\left(\frac{x-6}{2x+1}\right) = 0$

$10^0 = \frac{x-6}{2x+1}$

$1 = \frac{x-6}{2x+1}$

$2x+1 = x-6$

$x+1 = -6$

$x = -7$

but notice $\log(-7-6)$

$= \log(-13)$

which is NOT allowed

$\Rightarrow$ $\boxed{N.S.}$ (no solution)

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8. Perform the two stated row operations and then solve the system of equations. *(10 pts)*

$$x-2y+2z=9$$
$$-x+3y\quad=4$$
$$2x-5y+z=10$$

R_1+R_2 = $y+2z=13$

$-2R_1+R_3$ = $-y-3z=-8$

R_1+R_2:
$$\begin{array}{r} x-2y+2z=9 \\ +\ -x+3y\quad=4 \\ \hline y+2z=13 \end{array}$$

$-2R_1+R_3$:
$$\begin{array}{r} -2x+4y-4z=-18 \\ +\ 2x-5y+z=10 \\ \hline -y-3z=-8 \end{array}$$

finish:
$$\begin{array}{r} y+2z=13 \\ +\ -y-3z=-8 \\ \hline -z=5 \\ z=-5 \end{array}$$

$\Rightarrow y+2(-5)=13$
$y=23$

$\Rightarrow -x+3(23)=4$
$-x+69=4 \Leftrightarrow -x=-65 \Leftrightarrow x=65$

State the solution as an order triple: (x,y,z) $(65, 23, -5)$

9. For each of these, determine if there are many solutions, no solutions or one unique solution Show work for each. *(6pts)*

a. $2x-y=7$
$2\left(x+\frac{1}{2}y=-7\right)$

$$\begin{array}{r} 2x-y=7 \\ +\ 2x+y=-14 \\ \hline 4x=-7 \\ x=-7/4 \end{array}$$

$\Rightarrow$ one unique solution

b. $6x-9y=2$
$3\left(-2x+3y=-\frac{2}{3}\right)$

$$\begin{array}{r} 6x-9y=2 \\ +(-6x+9y=-2) \\ \hline 0=0 \end{array}$$

these are the same line

$\Rightarrow$ many solutions

c. $4x+3y=6$
$-2(2x+y=4)$

$$\begin{array}{r} 4x+3y=6 \\ -4x-2y=-8 \\ \hline y=-2 \end{array}$$

$\Rightarrow$ one unique solution

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10. Find the partial fraction decomposition of the rational expression: (*8 pts*)

$$\frac{x^2+3x+8}{x^3-4x}$$

$$= \frac{x^2+3x+8}{x(x^2-4)} = \frac{x^2+3x+8}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$\frac{x^2+3x+8}{x(x-2)(x+2)} = \frac{A(x-2)(x+2) + Bx(x+2) + Cx(x-2)}{x(x-2)(x+2)}$$

$$x^2+3x+8 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$$

$x=0$: $8 = A(-2)(2) \Rightarrow A = 8/4 = -2$

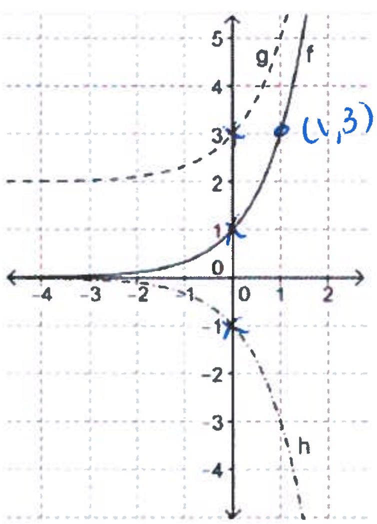
$x=-2$: $4-6+8 = C(-2)(-4) \Rightarrow 6 = 8C \Rightarrow C = 3/4$

$x=2$: $4+6+8 = B(2)(4) \Rightarrow 18 = 8B \Rightarrow B = \frac{9}{4}$

answer: $\boxed{\frac{-2}{x} + \frac{9/4}{x-2} + \frac{3/4}{x+2}}$

11. The graph of the function y = *f(x)* shown below is an exponential graph. The graphs of y = *g(x)* and y = *h(x)* are translations of y = *f(x)*. (*8 pts*)

$f(1) = 3 = a^1 = a \Rightarrow a = 3$



a. $f(x) = a^x$ What is *a*? 3

b. g(x) = $3^x + 2$ shifted up by 2

c. What is the asymptote of *g(x)*?
(*write as x=# or y=#*)

$y = 2$ (horizontal asymptote)

d. h(x) = -3^x
(a vertical reflection of $y = f(x)$)

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12. Answer the questions about the function and then sketch the graph. Show work to justify your answer. *(16 pts)*

$$f(x) = \frac{x^3+x^2-6x}{x^2+2x} = \frac{x(x^2+x-6)}{x(x+2)} = \frac{x(x-2)(x+3)}{x(x+2)}$$

Note: asymptotes should be given as lines, i.e. in the form y=# or x=#. Intercepts are points so should be given in the form (a,b). Also, "none" is a possible answer)

a. x-coordinate(s) of hole(s):

at $\boxed{x=0}$

(the factors that divide out of the function)

b. vertical asymptote(s):

$\boxed{x=-2}$

(since x+2 is still a factor of the denominator after simplifying)

c. horizontal or oblique asymptote:

$\boxed{\text{no HA}}$

$$\begin{array}{r} x-1-\frac{4}{x+2} \\ x+2 \,\big)\, x^2+x-6 \\ -(x^2+2x) \\ \hline -x-6 \\ -(-x-2) \\ \hline -4 \end{array}$$

$\Rightarrow$ oblique asymptote at $\boxed{y=x-1}$

d. x-intercept(s):

$0 = \frac{(x-2)(x+3)}{(x+2)}$

$0=(x-2)(x+3)$

$x=2$ or $x=-3$

$\boxed{\begin{matrix}(2,0)\\(-3,0)\end{matrix}}$

e. y-intercept

$y=\frac{(0-2)(0+3)}{(0+2)} = \frac{-2(3)}{2} = -3$

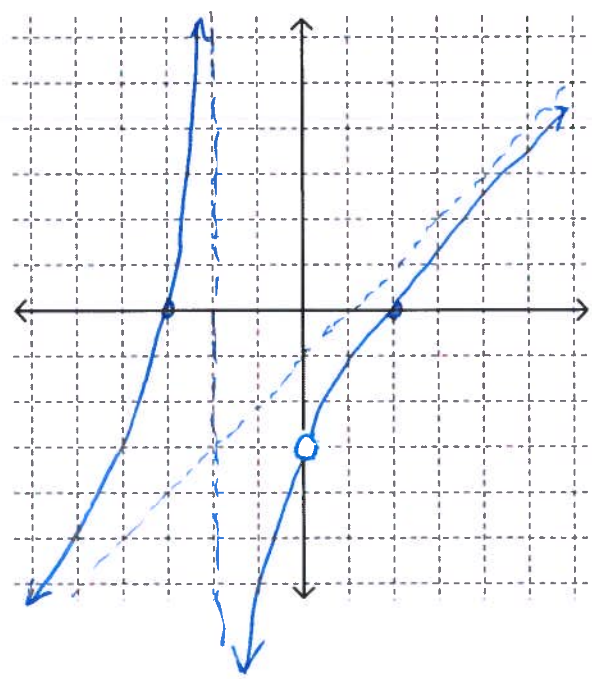
but there is a hole at x=0, which means we don't actually reach this pt

$\boxed{\text{none}}$

f. Graph the information from a-e and graph y = f(x).

Label your axes.

Note: if when you graph the information, you realize something is wrong, explain what you think is wrong and what a graph of a function like this looks like.



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