Solutions for practice in 2.1 Quadratic functions and models

1. Write an equations of the quadratic function with a vertex at (2,-3) which passes through the point (4,1)

$$y = a(x-2)^2 - 3$$

$(4,1)$:

$$1 = a(4-2)^2 - 3$$

$$1 = a4 - 3$$

$$4 = 4a$$

$$1 = a$$

$$\boxed{y = (x-2)^2 - 3}$$

2. Find the x and y intercepts and the vertex of this quadratic function: $y = x^2 - 2x - 3$.

y-intercept $x=0$:

$$y = 0^2 - 2\cdot 0 - 3 = -3$$

x-intercept $y=0$:

$$0 = x^2 - 2x - 3$$

$$0 = x - 3x + x - 3$$

$$0 = x(x-3) + (x-3)$$

$$0 = (x-3)(x+1)$$

$x = 3$ or $x = -1$

$(0,-3)$

$(3,0)$

$(-1,0)$

3. Find the x and y intercepts and the vertex of this quadratic function:
$y = -2x^2+8x+2$.

y-intercept $x=0$: $y=2$ $\boxed{(0,2)}$

x-intercept $y=0$: $-2x^2+8x+2=0 \quad / \div(-2)$

$x^2-4x-1=0$

$x_{1/2} = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} =$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$\boxed{(2+\sqrt{5}, 0) \quad (2-\sqrt{5}, 0)}$

$y = -2x^2+8x+2 =$

$= -2(x^2-4x)+2 =$

$= -2(x^2-4x+4-4)+2$

$= -2(x^2-4x+4)+8+2$

$= -2(x-2)^2+10 \Rightarrow$ vertex $\boxed{(2,10)}$

4. If the height of a ball thrown up in the air is given by this equation:
$h(t) = -16t^2+48t+160$
When does it hit the ground?
How high does it go?
When does it reach the highest point?

$h(t) = -16t^2+48t+160 = -16(t^2-3t)+160 =$

$= -16\left(t^2-3t+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)+160 =$

$= -16\left(\left(t-\frac{3}{2}\right)^2-\frac{9}{4}\right)+160 =$

$= -16\left(t-\frac{3}{2}\right)^2-16\cdot\left(-\frac{9}{4}\right)+160 =$

$= -16\left(t-\frac{3}{2}\right)^2+36+160 =$

$= -16\left(t-\frac{3}{2}\right)^2+196$

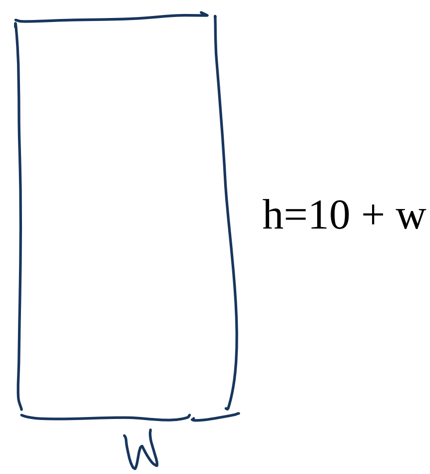
The vertex is at $\left(\frac{3}{2}, 196\right)$ so the highest point it gets to is at 196 meters, and it gets there after 1.5 seconds. To find when it hits the ground we have to find the roots:

$$-16t^2 + 48t + 160 = 0$$
$$t^2 - 3t - 10 = 0$$
$$t^2 - 5t + 2t - 10 = 0$$
$$t(t-5) + 2(t-5) = 0$$
$$(t-5)(t+2) = 0$$
$$t = -2, \quad t = 5$$

So the ball hits the ground after 5 seconds

5. If the area of a rectangle is 75 and the height is 10 more than the width, what are the dimensions of the rectangle?

$A = w \cdot h = 75$



$w \cdot (10 + w) = 75$

$10w + w^2 = 75$

$w^2 + 10w - 75 = 0$

$(w+15)(w-5) = 0$

$\cancel{w = -15}$, $w = 5$

Width is not negative

$h = 10 + 5 = 15$

$w = 5$

$h = 15$