

Solutions for practice in section 2.2 Polynomials

1. Determine the x-intercepts if possible, the y intercepts, and end behavior and sketch the function. If you cannot find the x-intercepts plot a few points to draw the graph.

$$F(x) = x^4 - 2x^3 - 3x^2$$

$$F(0) = 0 \quad (0, 0)$$

$$\begin{aligned} f(x) &= x^4 - 2x^3 - 3x^2 = \\ &= x^2(x^2 - 2x - 3) \\ &= x^2(x^2 - 3x + x - 3) = \\ &= x^2(x(x-3) + (x-3)) = \\ &= x^2(x+1)(x-3) \end{aligned}$$

$$\begin{aligned} x &= 0 \quad \text{double root} \\ x &= -1 \\ x &= 3 \end{aligned}$$

$$G(x) = -x^3 + 2x - 3$$

$$G(0) = -3 \quad (0, -3)$$

Cubic polynomial w leading coefficient < 0 , so we'll have

$$x \rightarrow -\infty \quad G(x) \rightarrow +\infty$$

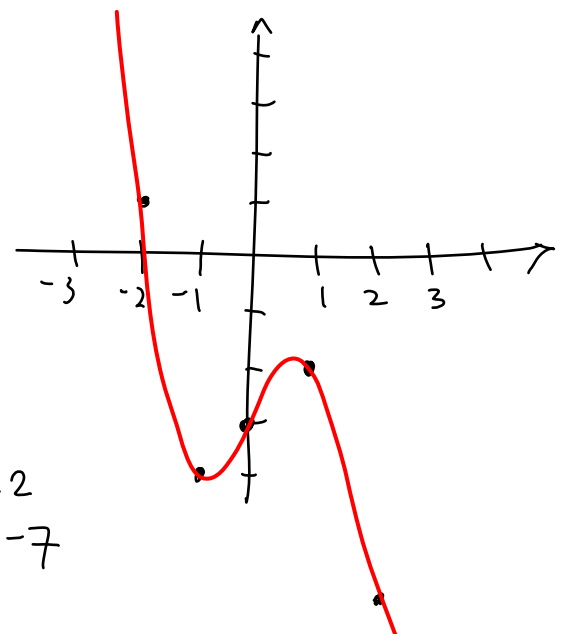
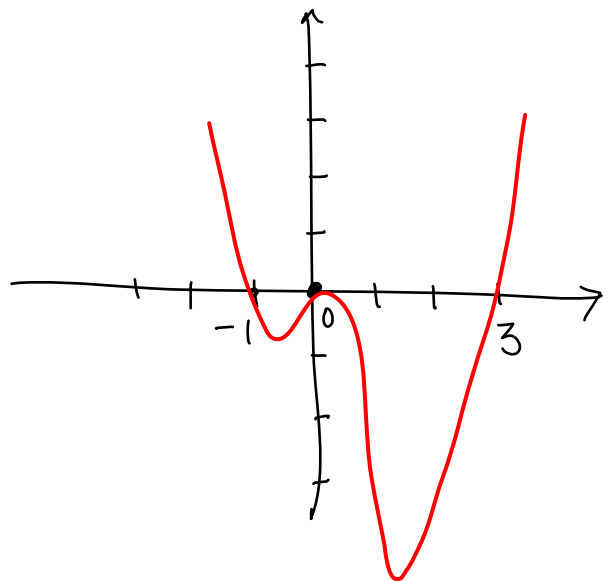
$$x \rightarrow +\infty \quad G(x) \rightarrow -\infty$$

x	G(x)
-3	$27 - 6 - 3 = 19$
-2	$8 - 4 - 3 = 1$
-1	$1 - 2 - 3 = -4$

x	G(x)
1	$-1 + 2 - 3 = -2$
2	$-8 + 4 - 3 = -7$

quartic function, leading coefficient > 0 , we have

$$\begin{aligned} x \rightarrow -\infty \quad F(x) &\rightarrow +\infty \\ x \rightarrow +\infty \quad F(x) &\rightarrow +\infty \end{aligned}$$



2. Sketch these transformations of $y = x^2$. Quick-draw them

a. $y = -2x^2$

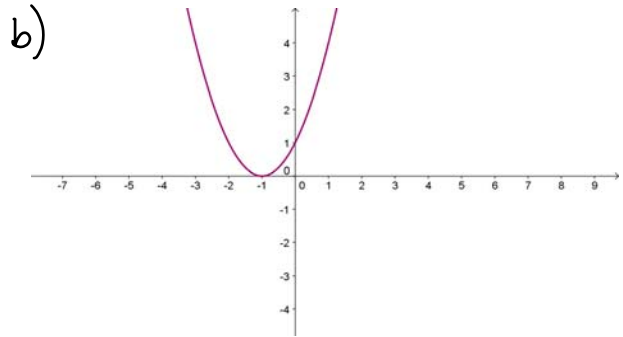
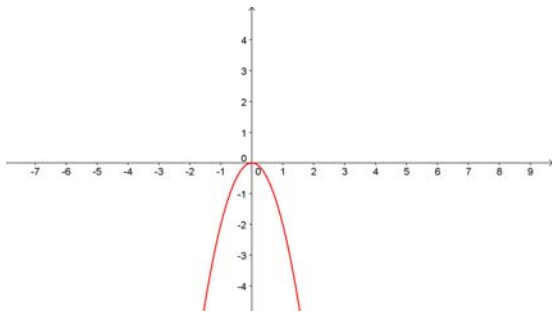
b. $y = (x+1)^2$

c. $y = x^2 - 3$

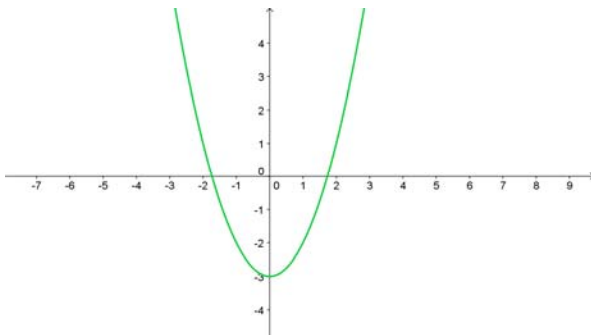
d. $y = -(x-2)^2 + 3$

e. $y = 0.5(x+2)^2 - 1$

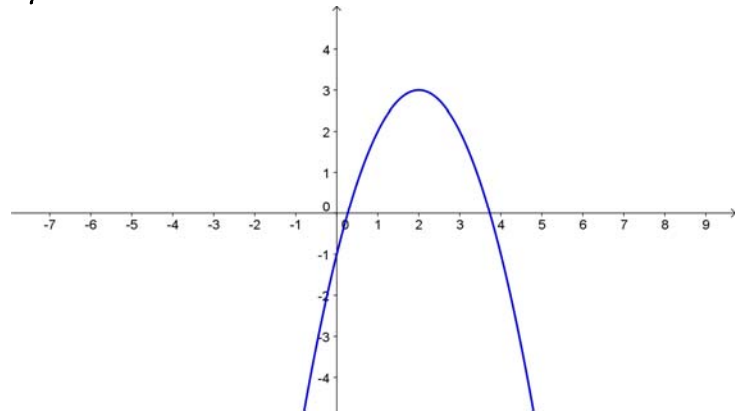
d. $y = -3x^2 - 2$



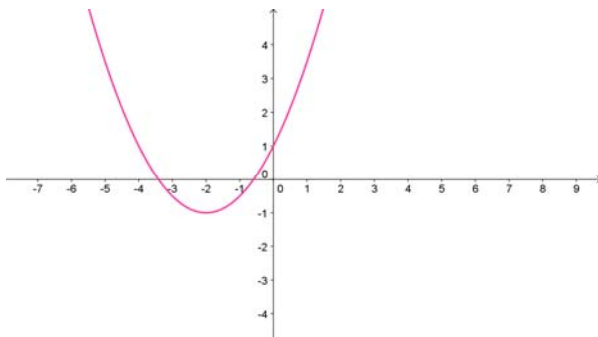
c)



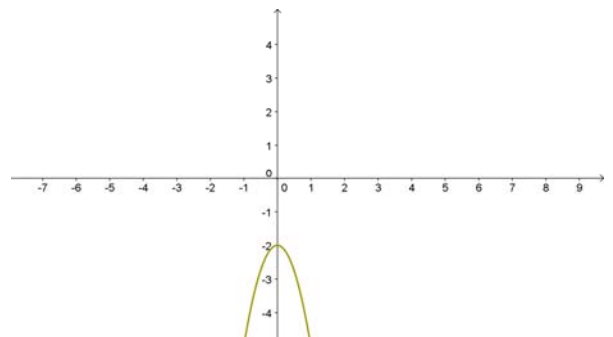
d)



e)



f)



3. Complete the square to sketch these quadratic functions using transformations. State the vertex, axis of symmetry and the x- intercepts and the y-intercept of each.

a. $y = 3x^2 - 6x + 2$

b. $y = -x^2 + 8x - 5$

$$\begin{aligned} \text{a)} \quad y &= 3x^2 - 6x + 2 = 3(x^2 - 2x + 1 - 1) + 2 = \\ &= 3(x^2 - 2x + 1) - 3 + 2 = 3(x-1)^2 - 1 \end{aligned}$$

Vertex : $(1, -1)$

axis : $x=1$

y-intercept : $(0, 2)$

x-intercept : $3(x-1)^2 - 1 = 0$

$$3(x-1)^2 = 1$$

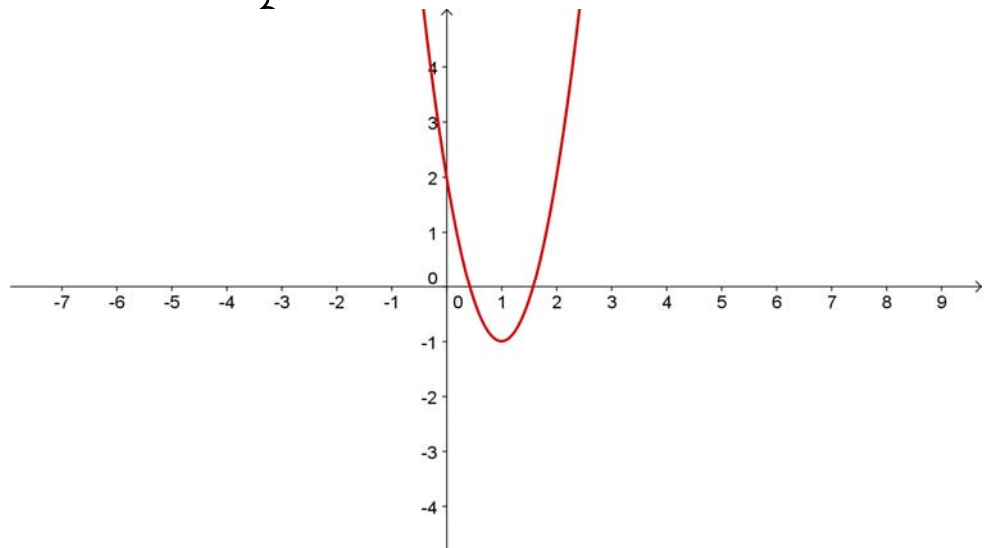
$$(x-1)^2 = \frac{1}{3}$$

$$x-1 = \pm \frac{\sqrt{3}}{3}$$

$$x = 1 \pm \frac{\sqrt{3}}{3}$$

$$\left(1 + \frac{\sqrt{3}}{3}, 0\right)$$

$$\left(1 - \frac{\sqrt{3}}{3}, 0\right)$$



b. $y = -x^2 + 8x - 5$

$$\begin{aligned} y &= -x^2 + 8x - 5 = -(x^2 - 8x + 16 - 16) - 5 = \\ &= -(x^2 - 8x + 16) + 16 - 5 = -(x-4)^2 + 11 \end{aligned}$$

Vertex: $(4, 11)$

axis: $x = 4$

y-intercept $(0, -5)$

x-intercept: $-(x-4)^2 + 11 = 0$

$$(x-4)^2 = 11$$

$$x-4 = \pm\sqrt{11}$$

$$x = 4 \pm \sqrt{11}$$

$$(4 + \sqrt{11}, 0)$$

$$(4 - \sqrt{11}, 0)$$

