Solutions for practice in 2.4 Complex numbers

1. Determine the sum, difference, product and quotient of these two complex numbers.

   3-2i 1+4i

$$(3-2i)+(1+4i)=3+1-2i+4i=4+2i$$

$$(3-2i)-(1+4i)=3-2i-1-4i=2-6i$$

$$(3-2i)(1+4i)=3+3\cdot 4i-2i\cdot 1-2i\cdot 4i=$$

$$=3+12i-2i-8(-1)=$$

$$=3+10i+8=11+10i$$

$$\frac{3-2i}{1+4i}\cdot\frac{1-4i}{1-4i}=\frac{3-12i-2i-8}{1+16}=\frac{-5-14i}{17}$$

2. Find the first six powers of this complex number: (2i)

$$(2i)^1=2i$$

$$(2i)^2=(2i)(2i)=4i^2=-4$$

$$(2i)^3=-4\cdot 2i=-8i$$

$$(2i)^4=-8i\cdot 2i=-16i^2=16$$

$$(2i)^5=16\cdot 2i=32i$$

$$(2i)^6=32i\cdot 2i=64i^2=-64$$

3. Find the value of: $(2-3i)^3$

$$(2-3i)^3 = (2-3i)(2-3i)(2-3i) = (2-3i)(4-6i-6i+9i^2) =$$

$$= (2-3i)(4-12i-9) = (2-3i)(-5-12i) =$$

$$= -10-24i+15i-36 =$$

$$= -46-9i$$

4. Find the value of: $(-i)^{53}$

$$(-i)^{53} = (-1)^{53} \cdot i^{53} = -i^{4\cdot 13+1} =$$

$$i^1 = i$$
$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

$$= -(i^4)^{13} \cdot i^1 =$$

$$= -(1)^{13} \cdot i = -i$$

5. Simplify and write in complex form: $\frac{2}{1-i} + \frac{3}{2+3i} =$

$$\frac{2}{1-i} \cdot \frac{1+i}{1+i} + \frac{3}{2+3i} \cdot \frac{2-3i}{2-3i} =$$

$$= \frac{2+2i}{2} + \frac{6-9i}{13} = \frac{13(2+2i)+2(6-9i)}{26} =$$

$$= \frac{26+26i+12-18i}{26} = \frac{38+8i}{26} = \frac{19+4i}{13}$$