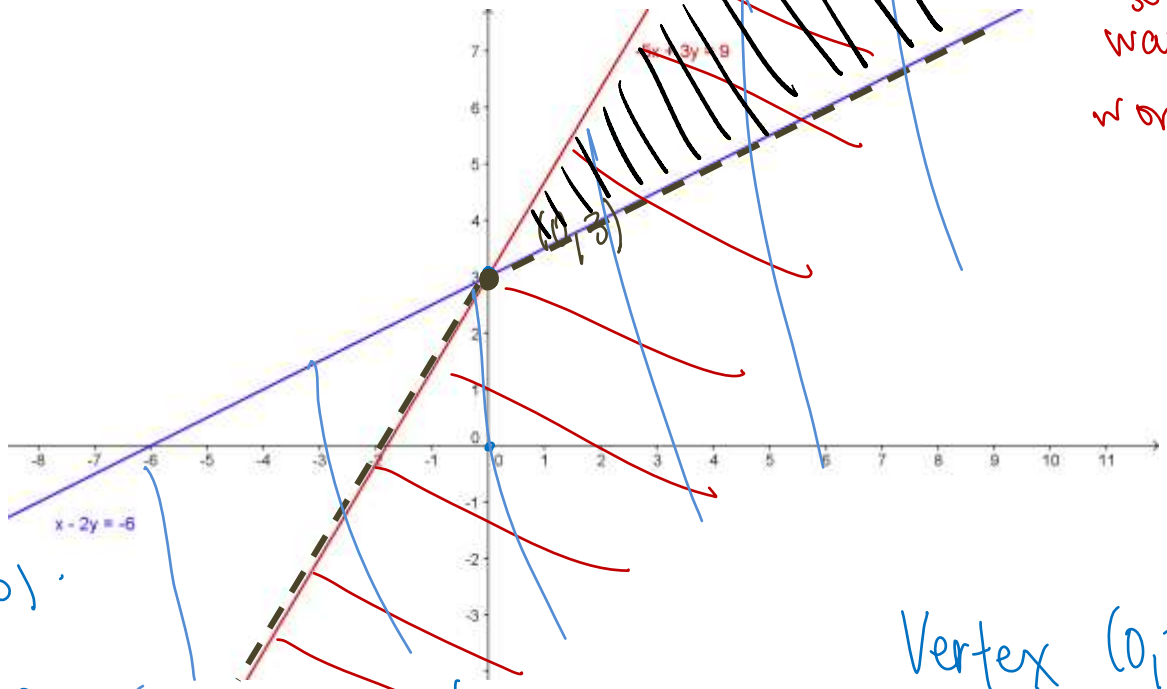
Solutions for practice in 7.5 Systems of Inequalities

Sketch the graph, label the vertices and shade in the appropriate area. These will be the constraints for 7.6 problems.

1. $x - 2y < -6$
   $5x - 3y > -9$



(0,0) : $-5 \cdot 0 + 3 \cdot 0 = 0 > -9$ so we want region w origin

(0,3)

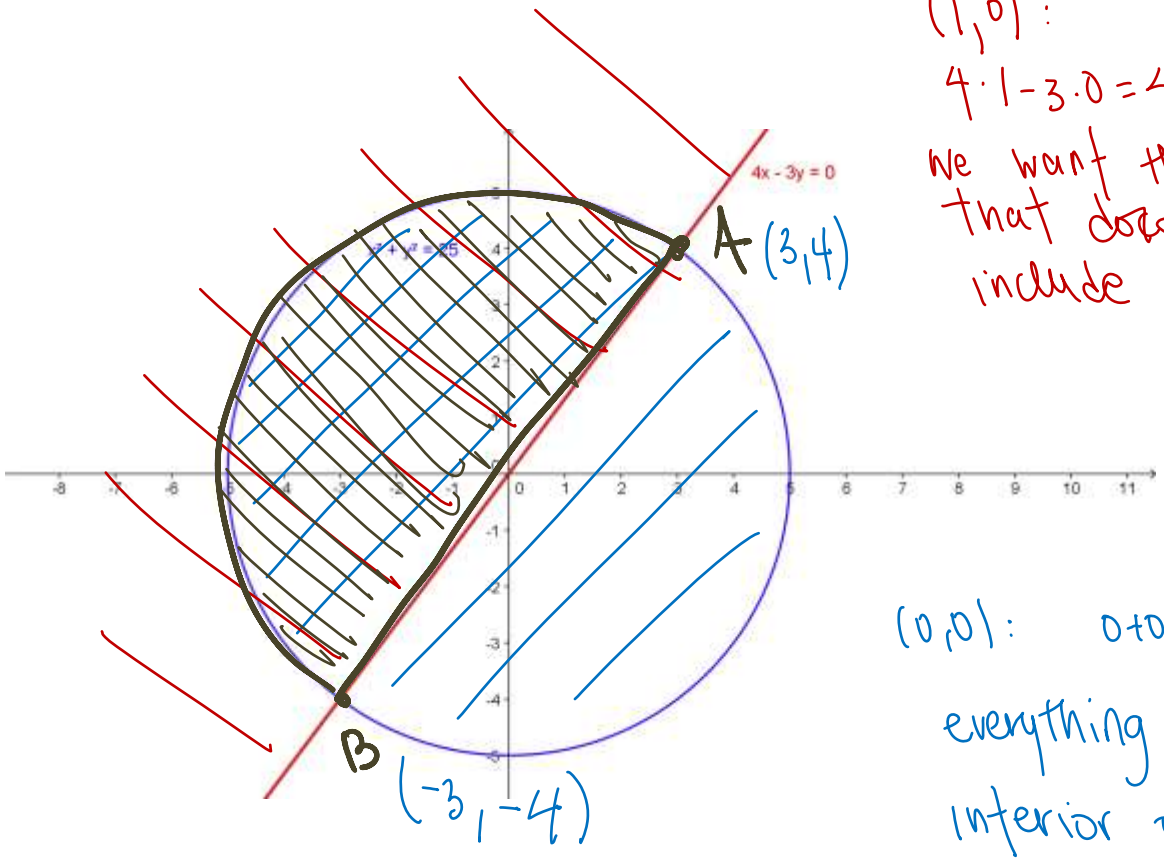
(0,0)

$0 - 2 \cdot 0 = 0 > -6$

we want the region w origin

Vertex (0,3)

2. $x^2 + y^2 \leq 25$
$4x - 3y \leq 0$



(1,0):
$4 \cdot 1 - 3 \cdot 0 = 4 > 0$
We want the side that does NOT include (1,0)

(0,0): $0+0 = 0 < 25$
everything in the interior of the circle & circle itself

$x^2 + y^2 = 25$
$4x - 3y = 0 \Rightarrow 4x = 3y \Rightarrow x = \frac{3y}{4}$

$\left(\frac{3y}{4}\right)^2 + y^2 = 25$
$\frac{9y^2}{16} + y^2 = 25$
$9y^2 + 16y^2 = 25 \cdot 16$
$25y^2 = 25 \cdot 16$
$y^2 = 16$

$y = \pm 4$

$y = 4 \quad x = \frac{3 \cdot 4}{4} = 3$

$y = -4 \quad x = -3$

A $(3,4)$
B $(-3,-4)$

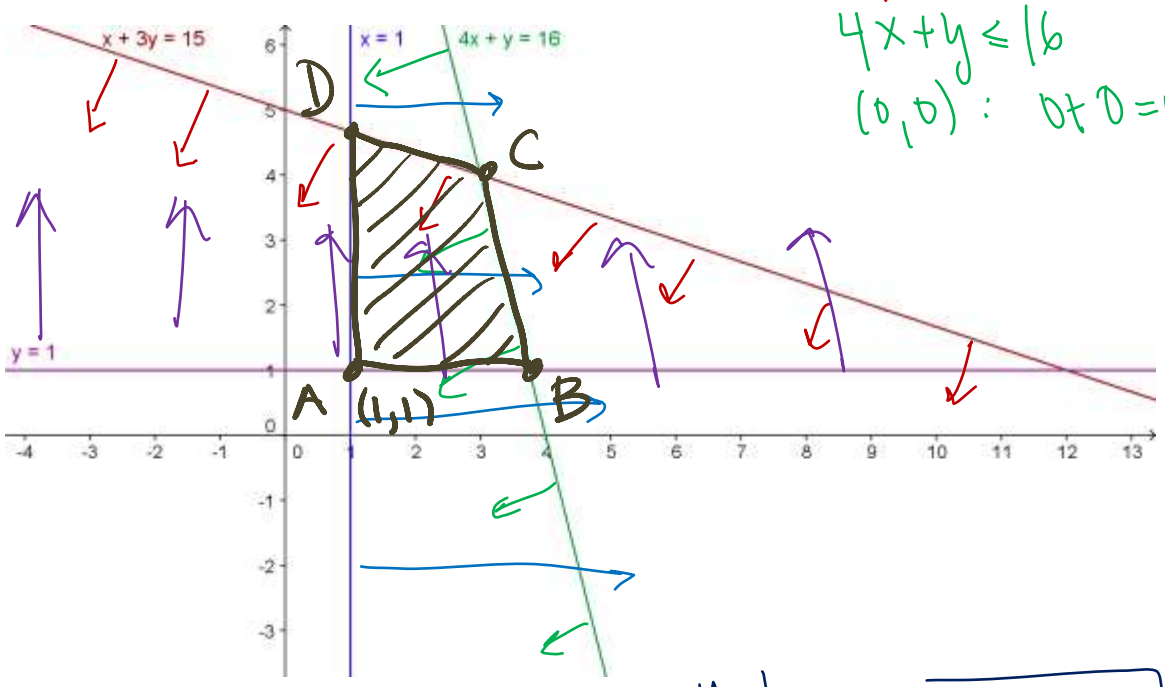
3. A toy shop produces widgets and gizmos. It takes 1 hour to make a widget and 4 hours to paint it. It takes 3 hours to make a gizmo and 1 hour to paint it. There are 15 hours available for construction and 16 for painting. The company must produce at least one of each toy.

x is number of widgets
y is number of gizmos

$x + 3y \le 15$
$4x + y \le 16$
$x \ge 1$
$y \ge 1$

we'll check $(0,0)$ for the first two equations

$x+3y \le 15$
$(0,0)$: $0+0=0 \le 15$
$4x+y \le 16$
$(0,0)$: $0+0=0 \le 16$



A: $x=1$, $y=1$ } $(1,1)$

B: $y=1$
$4x+y=16$
$4x+1=16$
$4x=15$
$\left(\frac{15}{4}, 1\right)$

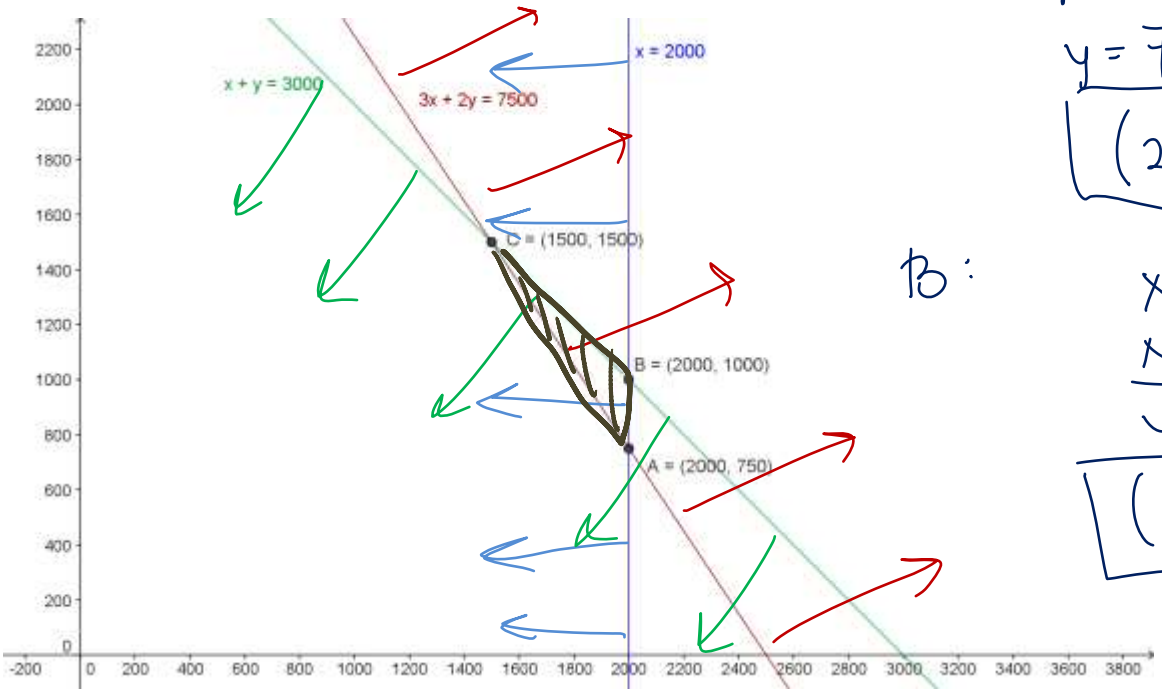
C: $x+3y=15 \Rightarrow x=15-3y$
$4x+y=16$
$4(15-3y)+y=16$
$60-12y+y=16$
$-11y=-44 \Rightarrow y=4$
$x=3$
$(3,4)$

D: $x=1$
$x+3y=15$
$1+3y=15 \Rightarrow 3y=14$
$\left(1, \frac{14}{3}\right)$

4. The concert (from lecture)
For a concert event, there are $30 reserved seat tickets, and $20 general admission tickets. There are 2000 reserved sets available and the fire regulations limit the number of paid ticket holders to 3000. The promoter must take in $75,000 in ticket sales. Find and graph the system of inequalities describing the different number of tickets that can be sold.

reserved tickets x
general admission tickets y

$x \geq 0$
$y \geq 0$
$x \leq 2000$
$x + y \leq 3000$
$30x + 20y \geq 75000$



A: $x = 2000$
$3x + 2y = 7500$
$6000 + 2y = 7500$
$2y = 1500$
$y = 750$
$(2000, 750)$

B:
$x = 2000$
$x + y = 3000$
$y = 1000$
$(2000, 1000)$

C: $x + y = 3000 \quad / \cdot (-3)$
$3x + 2y = 7500$
$-y = -1500$

$y = 1500$
$x = 1500$
$(1500, 1500)$