

Solutions for practice in 4.5 Linear Programming

Test the vertices from the previous constraint equations (from 4.4) in each of these objective equations and find the maximum and minimum for each.

1. $z = 2x - 3y$	2. $z = 3x + 2y$
3. Each gizmo sells for \$5.00 and each widget sells for \$4.00.	4. What is the maximum dollar amount they can take in at the concert?

1. Constraints equation:

$$\begin{aligned} x - 2y &< -6 \\ 5x - 3y &> -9 \end{aligned}$$

Objective equation:

$$z = 2x - 3y$$

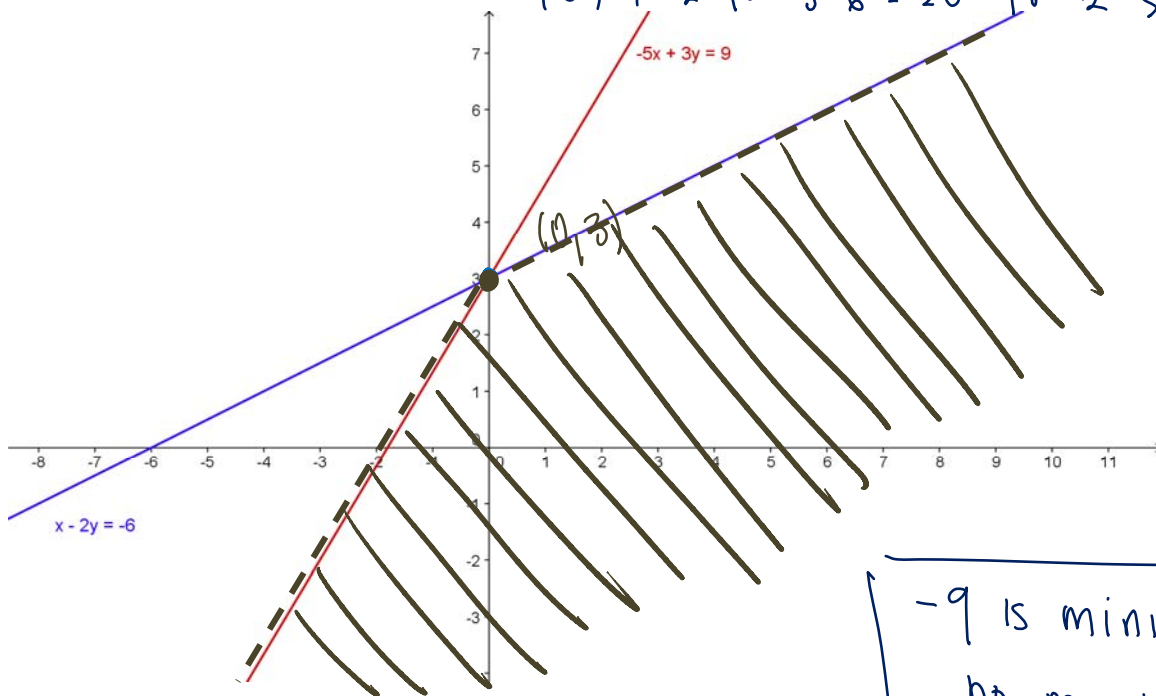
This is unbounded region and we should be careful:

Is $2x - 3y \geq -9$ on all points within region?

$$(-3, -4) : 2 \cdot (-3) - 3 \cdot (-4) = -6 + 12 = 6 > -9$$

$$(10, 6) : 2 \cdot 10 - 3 \cdot 6 = 20 - 18 = 2 > -9$$

$$\begin{aligned} x - 2y &< -6 \\ 5x - 3y &> -9 \end{aligned}$$



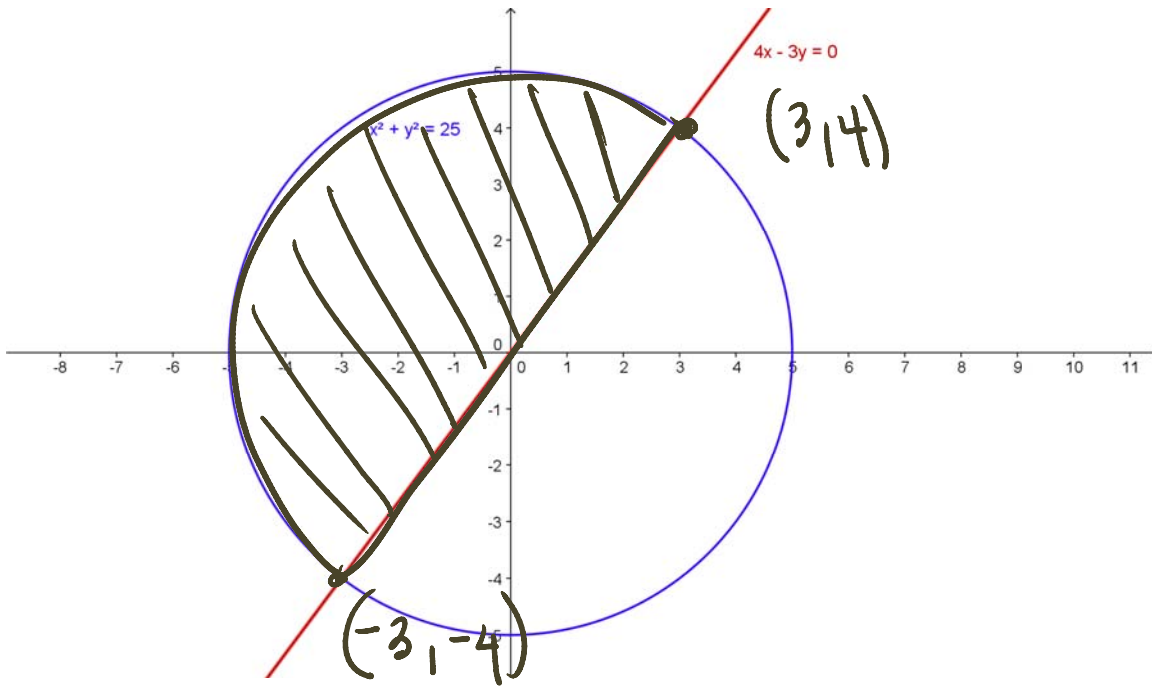
-9 is minimum
no maximum

2. Constraint equations

$$x^2 + y^2 \leq 25$$

$$4x - 3y \leq 0$$

Objective equation: $z = 3x + 2y$



$$z = 3x + 2y$$

$$(3, 4) : z = 3 \cdot 3 + 2 \cdot 4 = 9 + 8 = 17 \rightarrow \max$$

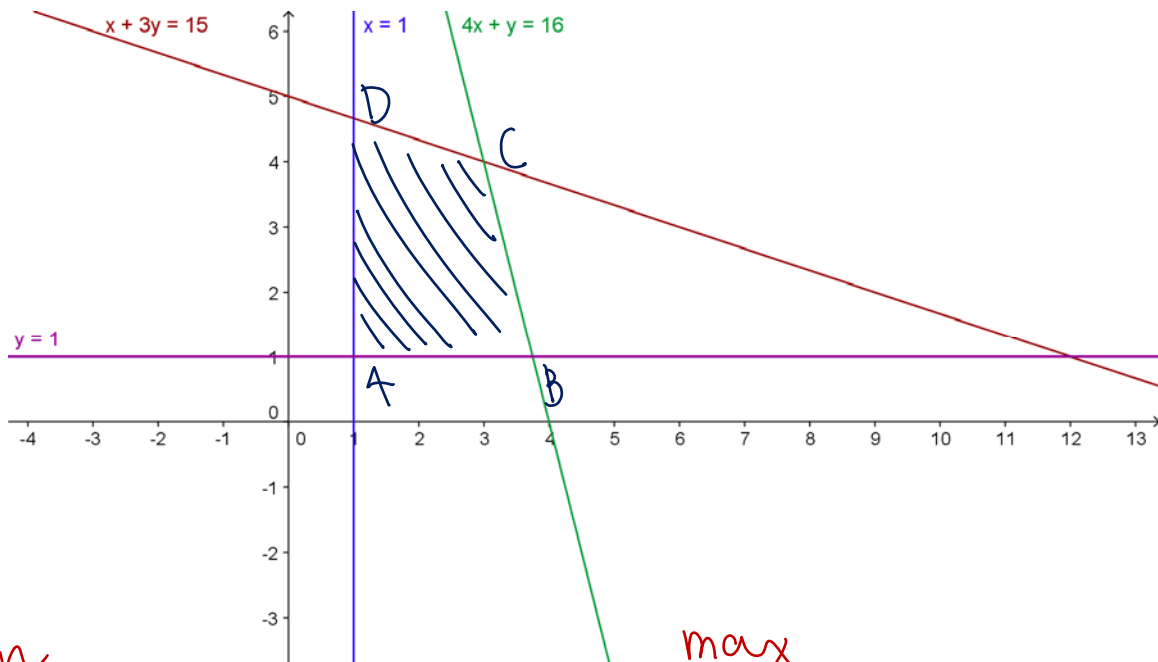
$$(-3, -4) : z = 3 \cdot (-3) + 2 \cdot (-4) = -9 - 8 = -17 \rightarrow \min$$

3. Each gizmo sells for \$5.00 and each widget sells for \$4.00.

x is number of widgets
y is number of gizmos

$$\begin{aligned} x + 3y &\leq 15 \\ 4x + y &\leq 16 \\ x &\geq 1 \\ y &\geq 1 \end{aligned}$$

$$z = 4x + 5y$$



min

$$A(1, 1) : z = 4 \cdot 1 + 5 \cdot 1 = 9$$

$$B\left(\frac{15}{4}, 1\right) : z = 4 \cdot \frac{15}{4} + 5 \cdot 1 = 20$$

max

$$C(3, 4) : z = 4 \cdot 3 + 5 \cdot 4 = 32$$

$$D\left(1, \frac{14}{3}\right) : z = 4 \cdot 1 + 5 \cdot \frac{14}{3} = \frac{12 + 70}{3} = \frac{82}{3}$$

$$\approx 27$$

4. What is the maximum amount they can take in at the concert?

reserved tickets x
 # general admission tickets y

$$x \geq 0$$

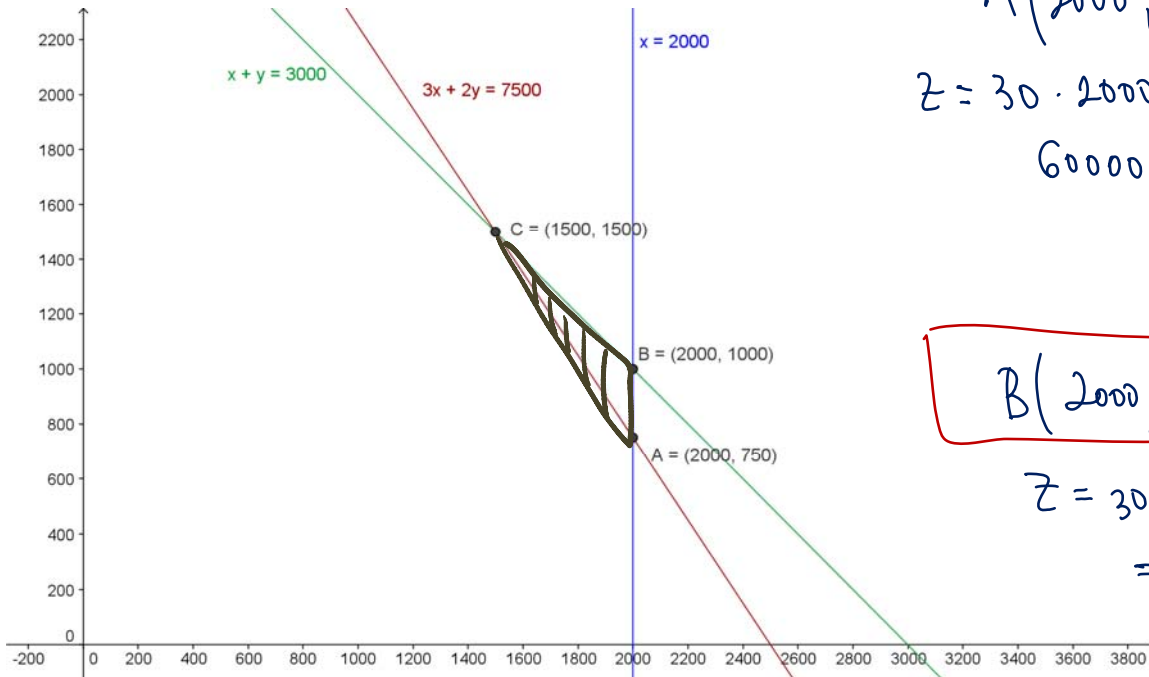
$$y \geq 0$$

$$x \leq 2000$$

$$x + y \leq 3000$$

$$30x + 20y \geq 75000$$

$$z = 30x + 20y$$



$$A(2000, 750)$$

$$z = 30 \cdot 2000 + 20 \cdot 750 = 60000 + 15000 = 75000$$

$$B(2000, 1000)$$

$$z = 30 \cdot 2000 + 20 \cdot 1000 = 60000 + 20000 = 80000$$

max

$$C(1500, 1500)$$

$$z = 30 \cdot 1500 + 20 \cdot 1500 = 45000 + 30000 = 75000$$

We know 75000 is the minimum.
 That was our constraint.