Solutions of practice problems for 5.1 Matrices and systems of equations

1. Complete this augmented matrix in row-echelon form to put it in reduced row-echelon form (zeros in the upper triangle.) Notice the last column of the matrix will be the solutions to the original problem. Check the notes from the lecture to verify.

$$\begin{bmatrix} 1 & 0 & -3 & : & -2 \\ 0 & 1 & 7 & : & 11 \\ 0 & 0 & 1 & : & 2 \end{bmatrix} \quad 1\cdot(-7) + \qquad 1\cdot 3 +$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

2. Write this system of equations as an augmented matrix and solve for x,y,z.

x + y – z = 0

2x - y = 4

-x - 2y + z = 2

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -1 & 0 & 4 \\ -1 & -2 & 1 & 2 \end{array}\right] \begin{array}{l} \cdot(-2) \quad \cdot 1 \\ + \\ + \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 2 & 4 \\ 0 & -1 & 0 & 2 \end{array}\right] \begin{array}{l} \\ \text{exch.} \\ /\cdot(-1) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & -3 & 2 & 4 \end{array}\right] \begin{array}{l} + \\ \cdot(-1) \quad \cdot(3) \\ + \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 2 & -2 \end{array}\right] \div(2) \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right] + \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x = 1$

$y = -2$

$z = -1$

Check:

$x + y - z = 0 \qquad 1 - 2 - (-1) = 1 - 2 + 1 = 2 - 2 = 0$ ✓

$2x - y = 4 \qquad 2 \cdot 1 - (-2) = 2 + 2 = 4$ ✓

$-x - 2y + z = 2 \qquad -1 - 2 \cdot (-2) + (-1) = -1 + 4 - 1 = -2 + 4 = 2$ ✓

3. When in row-echelon form one can detect systems which are dependent or inconsistent or have a unique solution. Solve each of these augmented matrices for (x,y,z) if possible. If not, state whether dependent or inconsistent.

1. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & -5 \end{array}\right]$ 2. $\left[\begin{array}{ccc|c} 1 & 4 & 7 & 10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 3 \end{array}\right]$ 3. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 7 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$ 4. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 0 \end{array}\right]$

1. Inconsistent! Last row claims $0=-5$.

2. will have a unique solution. let's check:

$$\left[\begin{array}{ccc|c} 1 & 4 & 7 & 10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} x=-1 \\ y=1 \\ z=1 \end{array}$$

3. is dependent. Last row says $0=0$, and second $y+7z=7 \Rightarrow y=7-7z$

First row claims $x=-2$.

Solutions can be written as: $\begin{bmatrix} -2 \\ 7-7z \\ z \end{bmatrix}, \ z \in \mathbb{R}$

4 has unique solution:

$x=6$
$y=7$
$z=0$, or $\begin{bmatrix} 6 \\ 7 \\ 0 \end{bmatrix}$.