Solutions for practice in 5.5 Applications of determinants.

1. This is a good time to demonstrate the many ways we have found in this chapter to solve a set of linear equations. Given this system:

$2x - 4y = 18$

$3x + y = 9$

a. Solve using Gaussian Elimination.

$$\left[\begin{array}{cc|c} 2 & -4 & 18 \\ 3 & 1 & 9 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 9 \\ 3 & 1 & 9 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 9 \\ 0 & 7 & -18 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cc|c} 1 & -2 & 9 \\ 0 & 1 & -18/7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 27/7 \\ 0 & 1 & -18/7 \end{array}\right]$$

$$x = \frac{27}{7}$$

$$y = -\frac{18}{7}$$

b. Solve by finding the inverse of the matrix of coefficients and using matrix algebra.

$$\left[\begin{array}{cc|cc} 2 & -4 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -2 & 1/2 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -2 & 1/2 & 0 \\ 0 & 7 & -3/2 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cc|cc} 1 & -2 & 1/2 & 0 \\ 0 & 1 & -3/14 & 1/7 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 1/14 & 2/7 \\ 0 & 1 & -3/14 & 1/7 \end{array}\right]$$

$$\begin{bmatrix} \frac{1}{14} & \frac{2}{7} \\ -\frac{3}{14} & \frac{1}{7} \end{bmatrix} \begin{bmatrix} 18 \\ 9 \end{bmatrix} = \begin{bmatrix} \frac{1}{\cancel{14}_7} \cdot \cancel{18}^9 + \frac{2}{7} \cdot 9 \\ -\frac{3}{\cancel{14}_7} \cdot \cancel{18}^9 + \frac{1}{7} \cdot 9 \end{bmatrix} = \begin{bmatrix} \frac{27}{7} \\ -\frac{18}{7} \end{bmatrix}$$

$$x = \frac{27}{7}$$

$$y = -\frac{18}{7}$$

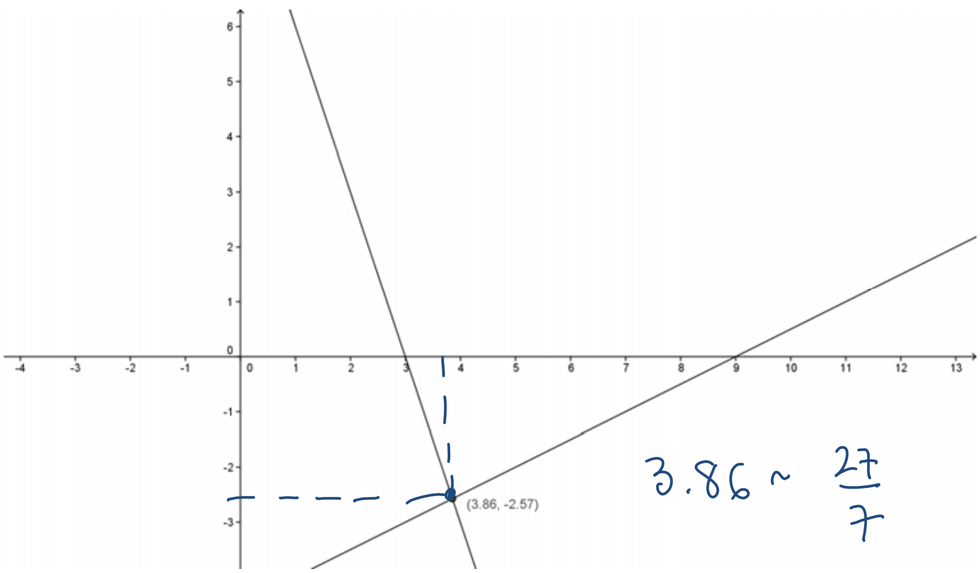
c. Solve using Cramer's rule: $2x - 4y = 18$

$3x + y = 9$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 18 & -4 \\ 9 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix}} = \frac{18 + 4 \cdot 9}{2 + 4 \cdot 3} = \frac{2(9+18)}{2(1+6)} = \frac{27}{7}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 2 & 18 \\ 3 & 9 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix}} = \frac{18 - 3 \cdot 18}{2 \cdot 7} = \frac{-2 \cdot 18}{2 \cdot 7} = -\frac{18}{7}$$

d. Solve by graphing



$3.86 \sim \frac{27}{7} = x$

$-2.57 \sim -\frac{18}{7} = y$

e. Solve by substitution

$$2x - 4y = 18$$
$$\underline{3x + y = 9}$$
$$x - 2y = 9$$
$$\underline{3x + y = 9}$$

$$x = 9 + 2y$$
$$3(9 + 2y) + y = 9$$
$$27 + 6y + y = 9$$
$$7y = -18$$
$$y = -\frac{18}{7}$$

$$x = 9 + 2 \cdot \frac{-18}{7} = \frac{63 - 36}{7} = \frac{27}{7}$$

$$x = \frac{27}{7}$$

I guess since I got the same answer EVERY time, it's prob. right :)

1. Use determinants to determine whether these three points are the vertices of a triangle. If they are, find the area of the triangle. If they are not, use determinants to write an equation of the line containing the three points.

   A (-2,9) B (2,1) C (4,-3)

$$D = \begin{vmatrix} -2 & 9 & 1 \\ 2 & 1 & 1 \\ 4 & -3 & 1 \end{vmatrix} = -2 - 6 + 36 - 4 - 6 - 18 =$$

$$\begin{matrix} -2 & 9 & 1 \\ 2 & 1 & 1 \end{matrix} \qquad = 0$$

These points are collinear:

$$\begin{vmatrix} -2 & 9 & 1 \\ 2 & 1 & 1 \\ x & y & 1 \end{vmatrix} = -2 + 2y + 9x - x + 2y - 18 = 0$$

$$\begin{matrix} -2 & 9 & 1 \\ 2 & 1 & 1 \end{matrix}$$

$$8x + 4y - 20 = 0$$

$$\boxed{4x + 2y - 10 = 0}$$