

Solutions for practice in 6.1 Sequences and Series

1. Write six terms in this sequence $a_n = \frac{(-2)^n}{n!}$

$$a_1 = \frac{(-2)^1}{1!} = \frac{-2}{1} = -2$$

$$a_2 = \frac{(-2)^2}{2!} = \frac{4}{2} = 2$$

$$a_3 = \frac{(-2)^3}{3!} = \frac{-8}{2 \cdot 3} = -\frac{4}{3}$$

$$a_4 = \frac{(-2)^4}{4!} = \frac{16}{2 \cdot 3 \cdot 4} = \frac{2}{3}$$

$$a_5 = \frac{(-2)^5}{5!} = \frac{-32}{2 \cdot 3 \cdot 4 \cdot 5} = \frac{-4}{15}$$

$$a_6 = \frac{(-2)^6}{6!} = \frac{2 \cdot 3 \cdot 4^2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{4}{45}$$

2. $\sum_{k=1}^4 (k-2)^2(k+1) =$

$$\begin{aligned} & (1-2)^2(1+1) + (2-2)^2(2+1) + (3-2)^2(3+1) + (4-2)^2(4+1) = \\ & = (-1)^2 \cdot 2 + 0 \cdot 3 + 1^2 \cdot 4 + 2^2 \cdot 5 = \\ & = 2 + 0 + 4 + 4 \cdot 5 = 26 \end{aligned}$$

3. Write a general term, a_n for this sequence 2, -4, 6, -8, ...

$$a_1 = 2$$

$$a_4 = -2 \cdot 4$$

$$a_n = (-1)^{n+1} \cdot 2n$$

$$a_2 = 2 \cdot 2$$

$$a_5 = 2 \cdot 5$$

$$a_3 = 2 \cdot 3$$

$$\text{check } a_6 = (-1)^7 \cdot 2 \cdot 6 = -12 \quad \checkmark$$

4. Use sigma notation to write the sum: $\frac{3}{1+2} + \frac{4}{2+3} + \frac{5}{3+4} + \dots =$

$$= \sum_{i=1}^{\infty} \frac{i+2}{i+(i+1)}$$

5. You see a sheet cake and decide to make it last forever... Each day you eat half of what is left. Use summation notation to indicate how much you ate. Will it ever be all gone?

$$\frac{1}{2} + \frac{1}{2} \left(\frac{1}{2} \right) + \frac{1}{2} \left(\frac{1}{4} \right) + \frac{1}{2} \left(\frac{1}{8} \right) + \dots =$$

\uparrow \uparrow \uparrow
 day 1 day 2 day 3
 ($\frac{1}{2}$ left) ($\frac{1}{4}$ left) ($\frac{1}{8}$ left)

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$= \sum_{i=1}^{\infty} \frac{1}{2^i}$$

Depends on what "ever" means 😊
 Not during your life time. Or your kids'. Or their kids'. Or theirs....