Solutions for practice in 6.3 Geometric sequences

1. Determine the tenth term and the sum of the first ten terms and the sum of an infinite number of terms for this geometric sequence: 3, 3/2, ¾, …

$a_1 = 3$

$a_2 = \frac{3}{2} = \frac{3}{2^1} = \frac{1}{2} \cdot 3$

$a_3 = \frac{3}{4} = \frac{3}{2^2} = \frac{1}{2^2} \cdot 3$

$\vdots$

$a_{10} = \frac{1}{2^9} \cdot 3$

$$S_{10} = a_1 \cdot \frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}} = 3 \cdot \frac{\frac{2^{10}-1}{2^{10}}}{\frac{1}{2}} = 3 \, \frac{2\left(2^{10}-1\right)}{2^{10}}$$

$$= 6 \, \frac{1023}{1024} \approx 5.99414$$

$$S_n = \frac{a_1}{1-r} = \frac{3}{1-\frac{1}{2}} = \frac{3}{\frac{1}{2}} = 6$$

2. $\sum_{k=1}^{7} -3(0.1)^k = \sum_{k=1}^{7} -3 \cdot \left(10^{-1}\right)^k = \sum_{k=1}^{7} -3 \cdot 10^{-k} =$

$$= -3 \cdot 10^{-1} - 3 \cdot 10^{-2} - 3 \cdot 10^{-3} - 3 \cdot 10^{-4} - 3 \cdot 10^{-5} - 3 \cdot 10^{-6} - 3 \cdot 10^{-7}$$

$$= \frac{-3}{10} - \frac{3}{100} - \frac{3}{1000} - \frac{3}{10000} - \frac{3}{100000} - \frac{3}{1000000} - \frac{3}{10000000}$$

$$= -0.3333333$$

3. $\sum_{j=1}^{\infty} 4\left(-\frac{2}{3}\right)^j$ $= a_1 \cdot \frac{1}{1-r} = -\frac{8}{3} \cdot \frac{1}{1+\frac{2}{3}}$

$a_1 = 4 \cdot \frac{-2}{3} = -\frac{8}{3}$

$= -\frac{8}{\cancel{3}} \cdot \frac{1}{\frac{5}{\cancel{3}}} =$

$r = -\frac{2}{3}$

$= -\frac{8}{5}$

4. In the example of the bouncing ball dropped from a height of 9 feet and bouncing up two-thirds of the previous distance on each bounce, what is the total distance it has traveled after bouncing ten times? If the ball could bounce indefinitely, what would be the distance traveled?

bounce # 1 : $\frac{2}{\cancel{3}} \cdot \cancel{9}\,3 = a_1 = 6$ $\quad r = \frac{2}{3}$

bounce #2 : $\frac{2}{3}\left(\frac{2}{3} 9\right) = a_2$

bounce #3 : $\frac{2}{3}\left(\frac{2}{3}\right)^2 9 = a_3$

$S_{10} = 6 \cdot \frac{1-\left(\frac{2}{3}\right)^{10}}{1-\frac{2}{3}} = 6 \cdot \frac{\frac{3^{10}-2^{10}}{3^{10}}}{\frac{1}{3}} = \overset{2}{\cancel{6}} \cdot \frac{3^{10}-2^{10}}{3^{\cancel{9}\,8}} = 2 \cdot \frac{59049-1024}{3^8}$

$= \frac{116050}{6561} = 17.6878$

but for each bounce the ball travels up & down the same distance & it travelled the 9 ft from the initial drop, so the total distance for 10 bounces:

$9 + 2 \cdot S_{10} = 9 + 2 \cdot 17.6878 = 44.3756$

$$S_\infty = a_1 \frac{1}{1-r} = \frac{6}{1-\frac{2}{3}} = \frac{6}{\frac{1}{3}} = 18$$

Similar thing happens here. We accounted only for ups. Add all downs:

$$9 + 2 \cdot S_\infty = 9 + 2 \cdot 18 = 9 + 36 = 45$$

5. Now you get really brave in saving for your trip and each day you deposit twice the amount you did on the previous day, starting with \$1.00 on day 1. How much will you deposit on the 30th day? What is the total amount in the account on day 30?

$a_1 = 1$

$r = 2$

$$a_{30} = a_1 \cdot r^{30-1} = 1 \cdot 2^{29} = 536870912$$

$$S_{30} = a_1 \frac{1-r^{30}}{1-r} = \frac{1-2^{30}}{1-2} = 2^{30} - 1$$

$$= 1073741824$$

Yeah, right!