9.5 The Binomial Theorem *Solution*

- Use the Binomial Theorem to calculate binomial coefficients.
- Use Pascal's Triangle to expand and calculate binomial coefficients
- Find the nth term in a binomial expansion.

Pascal:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

1. Use the Binomial Theorem or Pascal's Triangle to expand this: $(2x-3y)^5$

$$1(2x)^5 + 5(2x)^4(-3y) + 10(2x)^3(-3y)^2 + 10(2x)^2(-3y)^3 + 5(2x)(-3y)^4 + 1(-3y)^5$$

$$= 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5$$

2. Determine the coefficient of the term which has x^3y^4 in the expansion of $(x-2y)^7$.

$$\binom{7}{4}(x)^3(-2y)^4 = \frac{7!}{3!4!}\cdot(16)x^3y^4 = \frac{7\cdot\cancel{6}\cdot5}{\cancel{3\cdot2}}(16)x^3y^4 = 560x^3y^4$$

Coeff: $\underline{\underline{560}}$

10th Row $\binom{10}{0}\binom{10}{1}\binom{10}{2}\cdots$

3. Determine the 8^{th} term of this expansion $(4x+3y)^{10}$.

Remember first term is $\binom{10}{0}$ Second is $\binom{10}{1}$

So 8^{th} is $\binom{10}{7}$

$$\binom{10}{7}(4x)^7(3y)^3 = \frac{10\cdot9\cdot8}{3\cdot2}4^7\cdot3^3x^7y^3 = 53{,}084{,}160x^7y^3$$

For more practice work from exercises 15-53 in the text.