Non-rigid transformations

In non-rigid transformations, the shape of a function is modified, either “stretched” or “shrunk”. We will call the number which tells us how much it is changed the “scale factor”, and often use the letter “a” to describe it in general equations. Here are some examples:

Parent function	New function	Scale factor
$f(x) = x^2$	$g(x) = a(x-h)^2 + k$	a
$f(x) = x^2$	$g(x) = 3x^2$	3
$f(x) = \|x\|$	$g(x) = 0.5\|x\|$	0.5
$f(x) = \sqrt{x}$	$g(x) = 2\sqrt{x}$	2

The scale factor is a multiplier. It affects the y-coordinates of points on the parent graph. Consider the following:

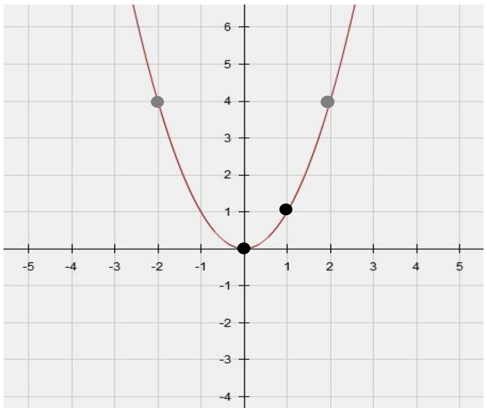
Points on parent function $f(x) = x^2$	Corresponding points on new function $g(x) = 3x^2$
(-2,4)	(-2,12)
(-1,1)	(-1,3)
(0,0)	(0,0)
(1,1)	(1,3)
(2,4)	(2,12)

In general, if *a* is the scale factor and (x,y) is a point on the parent function, the corresponding point on the new function is (x,a·3). For example, in the table above, the point (-2,4) corresponds to the point (-2,12) because 3·4=12.

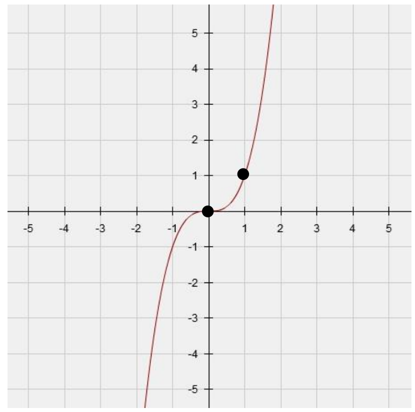
One of the easiest ways to graph or to identify a non-rigid transformation is by plotting “key points”. We are first interested in points that do not move under a stretch or a shrink. When looking at the parent graphs, these points are always located on the x-axis. (Since their y-coordinate is 0, they are unchanged by multiplication.) We will call such points “**anchor points”.**

If a point on a graph isn’t an anchor point, then it is either stretched or shrunk under a non-rigid transformation. We could figure out the scale factor by looking at what happens to any point. We will choose the easiest one(s) to follow, and label them “**moving points**”. For many of the graphs that we work with, the moving point is one unit to the right and one unit above the anchor point. In a graph that has had a transformation, the moving point is still one unit to the right, but its vertical distance from the anchor point equals the scale factor. In a few cases we give alternative moving points. Finding the scale factor with them involves more calculating, but if a graph is shrunk, it may be easier to accurately see where these points go.

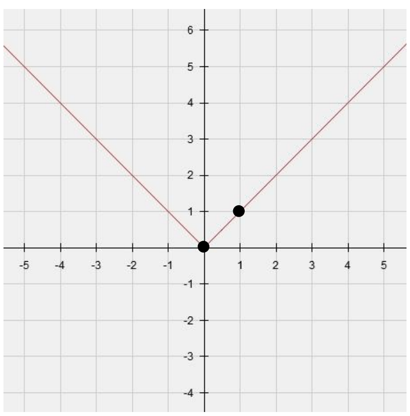
These are the anchor and the moving points for the parent functions we work with:



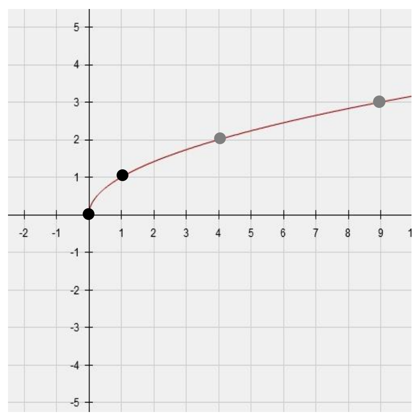
Quadratic (parabola):
ANCHOR POINT: vertex (0,0)
MOVING POINT: point 1 to the right of the vertex (1,1)



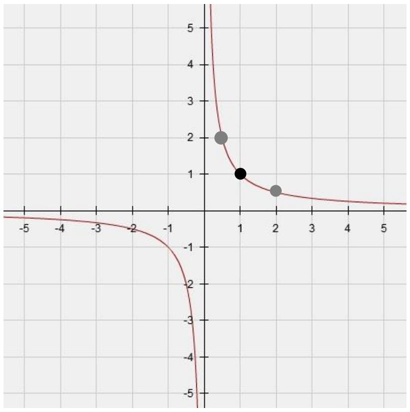
Cubic::
ANCHOR POINT: point of inflection (turning point) (0,0)
MOVING POINT: point 1 to the right of the vertex (1,1)



Absolute Value:
ANCHOR POINT: point at tip (0,0)
MOVING POINT: point 1 to the right of the tip (1,1)



Square root:
ANCHOR POINT: endpoint (0,0)
MOVING POINT: point 1 to the right of the endpoint (1,1)
OTHER MOVING POINT: point 4 to the right of the endpoint (4,2)
OTHER MOVING POINT: point 9 to the right of the endpoint (9,3)

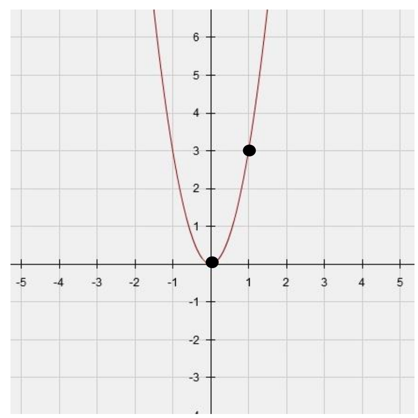


The reciprocal graph is an exception. It doesn't have any anchor points. However, its asymptotes (which we study more in later sections) behave a bit like the anchor points.

Reciprocal (hyperbola):
MAIN MOVING POINT: point on the graph 1 unit right of the vertical asymptote and 1 unit up from the horizontal asymptote.
OTHER MOVING POINT: point 1/2 to the left of the main point
OTHER MOVING POINT: point 1 to the right of the main point

We now give examples of looking at graphs and determining if they have been stretched and shrunk and what their scale factor is.

Example 1: This parabola has been stretched. The moving point is now 3-units away (vertically) from the anchor point. The scale factor is 3 and the new function graph is $y=3x^2$.

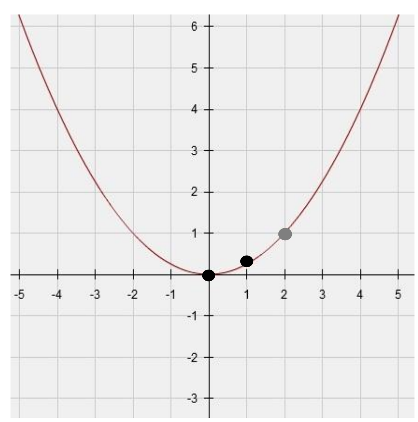


Example 2: This parabola has been shrunk. The moving point is now ¼ units away (vertically) from the anchor point. However, depending on how big the graph is and how good your eyes are, this may be hard to see. We look at a second moving point, the point (2,4) on the parent graph. It is now at (2,1). We set up an equation using the y-values. (Let a be the scale factor we are trying to find.)

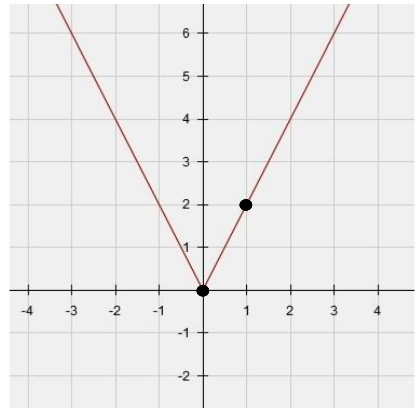
$$4 \cdot a = 1$$

$$a = \frac{1}{4}$$

The new function graph is $y = \frac{1}{4}x^2$.

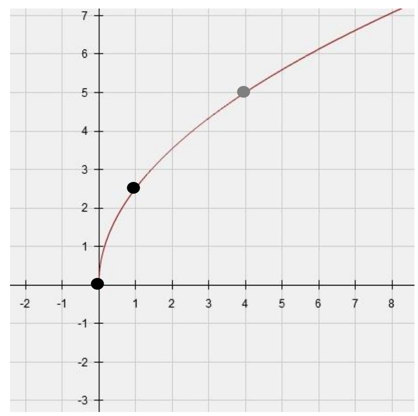


Example 3: This absolute value graph has been stretched. The moving point is now 2 units away (vertically) from the anchor point.



The new function graph is $y = 2|x|$.

Example 4: This square root graph has been stretched. The moving point is now 2.5 units away (vertically) from the anchor point. We could check this using a second moving point. The point at (4,2) from the parent graph is now at (4,5).

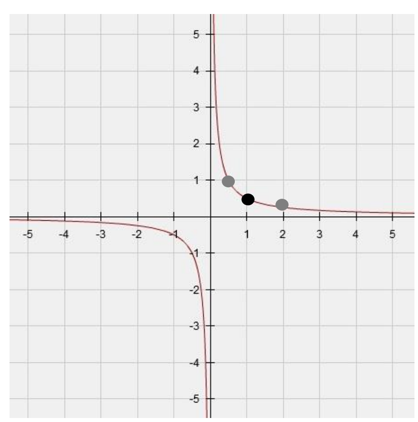


$$2 \cdot a = 5$$

$$a = \frac{5}{2}$$

The new function graph is $y = \frac{5}{2}\sqrt{x}$.

Example 5: This reciprocal graph has been shrunk. The moving point has gone from (1,1) to what looks like (1,1/2). This indicates a scale factor of ½. We can check it using the moving points to the left of the first, since it's the easiest to see. It was at (1/2, 2) and goes to (1/2,1). The moving point to the right of the first is hard to see. The scale factor is:



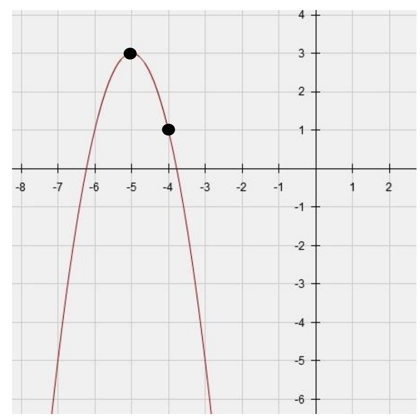
$$2 \cdot a = 1$$

$$a = \frac{1}{2}$$

The new function graph is $y = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$.

We can also use this technique on graphs that have undergone rigid transformations (moving vertically, horizontally, or being reflected.)

Example 6: This parabola has been stretched. The vertical difference between the anchor point and the moving point is -2, so the scale factor is -2. (Or just 2 and then we add the "-" because of the reflection).



The vertex has also moved to (-5,3), so h = -5 and k =3.
Thus, we can replace values in the standard from of the parabolic equation,

$$y = a(x-h)^2 + k$$

$$y = -2(x-(-5))^2 + 3$$

$$y = -2(x+5)^2 + 3$$