

### Graphing with Points

One of the disadvantages of an online course is that the online format makes it difficult for you to get feedback on graphing functions. As an instructor, unless you are visiting on office hours or sending me scanned graphs in e-mails, I may not get a chance to comment on you graphing skills until the exam (at which time I not only need to comment, but also to grade them.)

This worksheet is designed to help graph accurately and to let you know what I will be looking for when I grade graphing problems on exams.

#### Sketching versus graphing a function

Sketching a function – less accurate, shows key features of a graph like its general form (is it a quadratic or a cubic), whether increasing or decreasing, or how many intercepts it has. Great if this general information is what you need to solve the problem. (Often useful with higher degree polynomials)

Graphing a function - accurate, graph must pass through correct points. The number of key points and how to determine them depends upon the function (use table, use transformations, etc.)

These are the “key points” that are important for accuracy when graphing functions. For information about HOW to graph, refer to the video lectures and the textbook.

#### Horizontal line or vertical line:

- One point on the line (intercepts are often chosen)

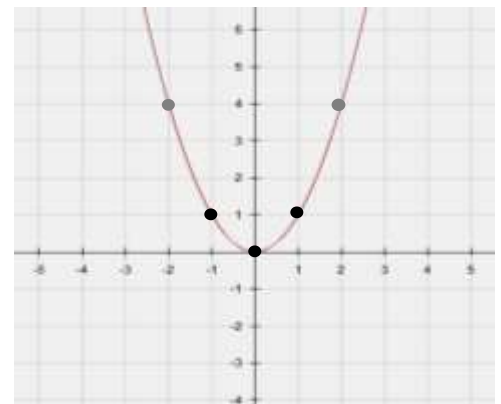
#### All other lines: (two points on the line)

- |  |    |   |    |   |
|--|----|---|----|---|
| <ul style="list-style-type: none"> <li>• x-intercept</li> <li>• y-intercept</li> </ul> | or | (Efficient if function is in<br>$y = mx + b$ form) <ul style="list-style-type: none"> <li>• y-intercept</li> <li>• point determined from the slope</li> </ul> | or | <ul style="list-style-type: none"> <li>• Any two points – you choose</li> </ul> |
|--|----|---|----|---|

#### Quadratic (Parabola, parent function $y = x^2$ shown)

(Efficient if function is in  $y = a(x - h)^2 + k$  form)

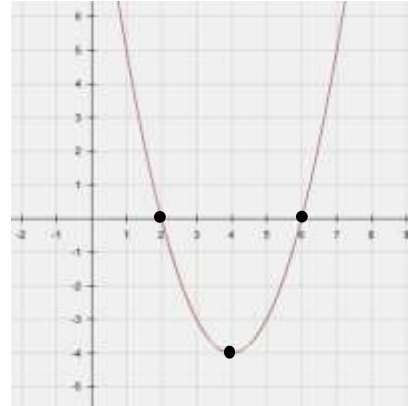
- Vertex
- Points  $\pm 1$  unit horizontally from the vertex,
- (maybe) points  $\pm 2$  unit horizontally from the vertex



Or

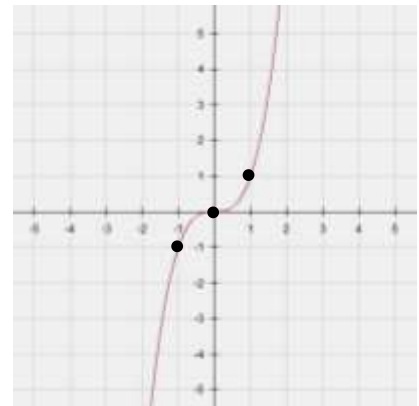
(Efficient if function has two real roots – i.e. two x-intercepts- and is in  $y = a(x - r_1)(x - r_2)$  form)

- One x-intercept  $(r_1, 0)$
- Other x-intercept  $(r_2, 0)$
- Vertex



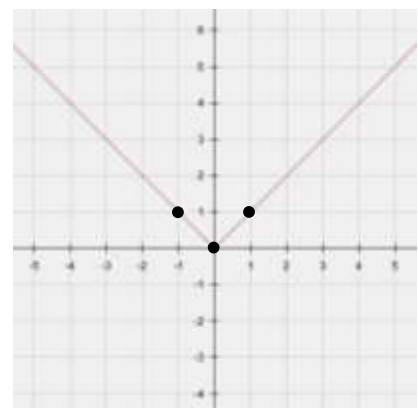
**Cubic** (parent  $y = x^3$ )

- Point of inflection (turning point),
- Points  $\pm 1$  unit horizontally from the point of inflection
- (maybe – not shown) Points  $\pm 2$  unit horizontally from the point of inflection



**Absolute Value** (parent  $y = |x|$ )

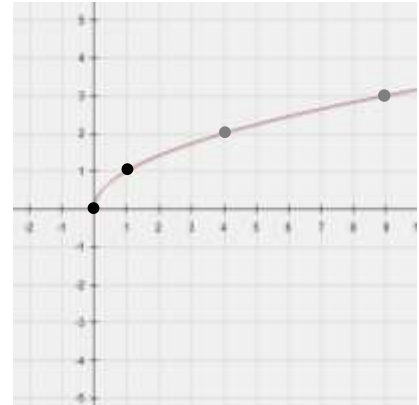
- Point at tip  $(0, 0)$
- Points  $\pm 1$  unit horizontally from the tip



### Square Root

(parent  $y = \sqrt{x}$ )

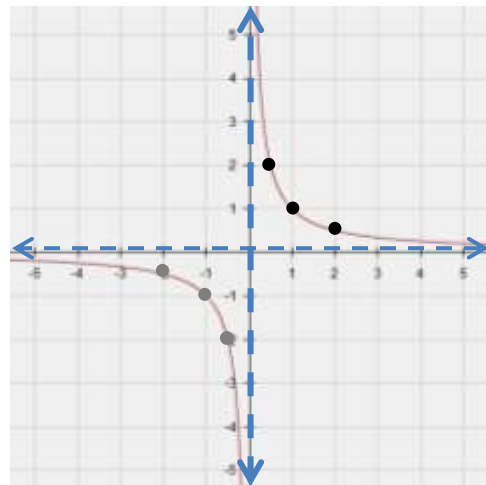
- Point at end (0,0)
- Points +1 unit horizontally from the tip
- (maybe) Point +4 units horizontally from the end (because  $\sqrt{4}$  is a whole number)
- (maybe) Point +9 units horizontally from the end (because  $\sqrt{9}$  is a whole number)



### Reciprocal (hyperbola)

(parent  $y=1/x$ ; general  $y=a(1/x)$ )

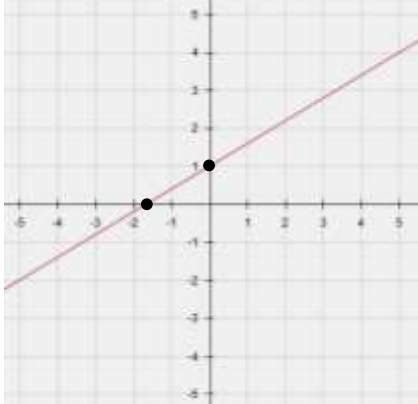
- Vertical asymptote ( $x=0$ )
- Horizontal asymptote ( $y=0$ )
- point on graph 1 unit right of the vertical asymptote
- point on graph 2 units right of the vertical asymptote
- (maybe)one more point if the shape of the curve is not yet clear
- Two to three point on the other curve (Note: if a point on the first curve is x-units right and y-units up from the asymptotes, then the corresponding point on the other curve will be x-units left and y-units down)



### Rational Functions of the form $y = \frac{ax+b}{cx+d}$

Not in a form where we can use transformations. Instead, find vertical and horizontal asymptotes and then make a table to determine nearby points. (See Example 7)

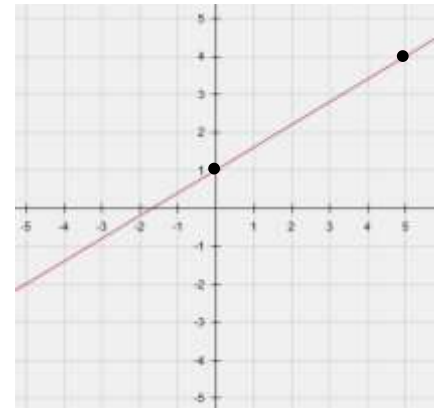
## Examples

Example 1:  $y = 3/5x + 1$ 

(Graph on left)

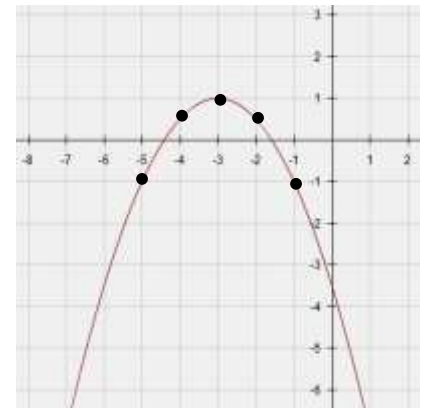
Using x-intercept:  $(-5/3, 0)$   
and y-intercept:  $(0, 1)$ 

(graph on right)

Using y-intercept  $(0, 1)$   
and point determined from slope: start  
at  $(0, 1)$  move 3 units up and 5 units right;  
arrive at  $(5, 4)$ Example 2: Parabola  $y = -\frac{1}{2}(x + 3)^2 + 1$ 

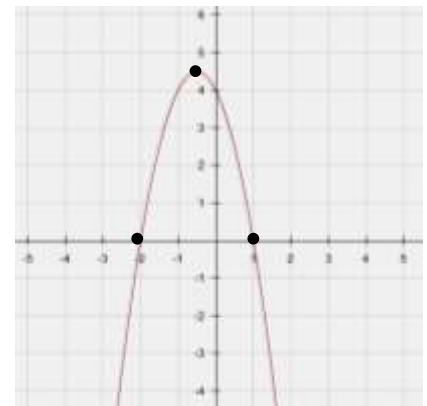
Points:

- Vertex:  $(-3, 1)$
- Points  $\pm 1$  unit horizontally from the vertex:  
 $(-4, \frac{1}{2})$  and  $(-2, \frac{1}{2})$   
(The corresponding points on the parent graph are 1 unit above the vertex. These points are  $\frac{1}{2}$  units lower, because the graph is shrunk by a factor of  $-1/2$ .)
- points  $\pm 2$  unit horizontally from the vertex:  
 $(-5, -1)$  and  $(-1, -1)$   
(The corresponding points on the parent graph are 4 units above the vertex. These points are 2 units lower, because the graph is shrunk by a factor of  $-1/2$  and  $2=4 \cdot (-1/2)$ .)

Example 3: Parabola  $y = -2(x + 2)(x - 1)$ 

Points:

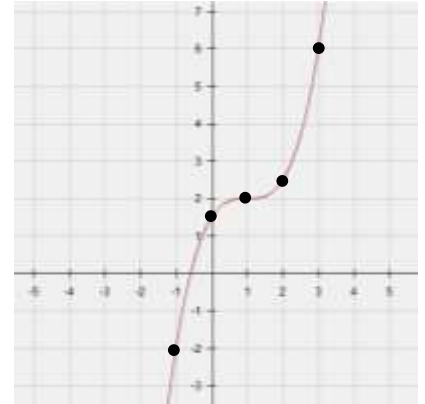
- One x-intercept :  $(-2, 0)$
- Second x-intercept :  $(1, 0)$
- Vertex:  $(-1/2, 9/2)$



Example 4: Cubic  $y = \frac{1}{2}(x - 1)^3 + 2$

Points:

- Point of inflection (1,2)
- Points  $\pm 1$  unit horizontally away: (0,3/2), (2,5/2)  
(Vertical distance is  $\frac{1}{2} \cdot 1 = 1/2$ )
- Because the scale factor is less than one, the points  $\pm 2$  units horizontally away are on the graph: (-1,-2), (3,6)  
(Vertical distance is  $\frac{1}{2} \cdot 8 = 4$ )



Example 5: Absolute value  $y = \frac{1}{3}|x + 3| - 2$

Points:

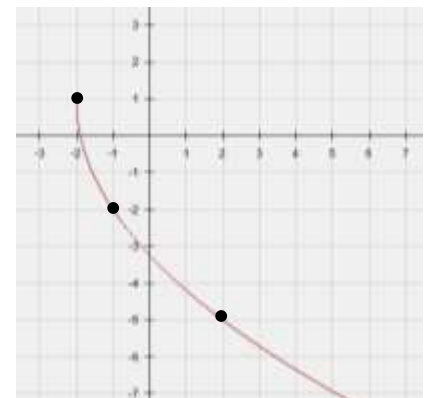
- Point at tip (-3,-2)
- Points:  $(-4, 1\frac{2}{3}), (-2, 1\frac{2}{3})$  (Vertical distance is  $\frac{1}{3} \cdot 1 = 1/3$ )
- (Alternative) you can use the slope, "over three and up one" in both directions from tip: (-6,-1), (0,-1)



Example 6: Absolute value  $y = -3\sqrt{x + 2} + 1$

Points:

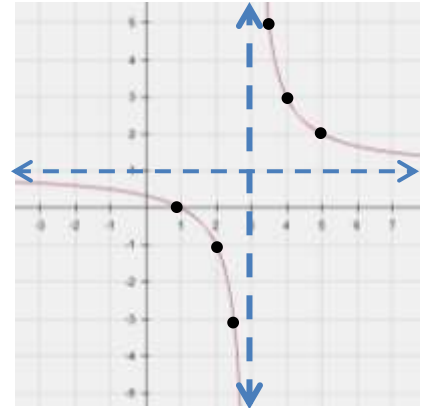
- Point at end (-2,1)
- Point +1 unit horizontally (-1,-2)  
(Vertical distance is  $-3 \cdot 1 = -3$ )
- Points +4 units horizontally from the end (0,-5)  
(Vertical distance is  $-3 \cdot 2 = -6$ )



Example 7: Reciprocal  $y = \frac{2}{x-3} + 1$

Points:

- Vertical asymptote  $(x+3)$  (Shifted right by 3)
- Horizontal asymptote  $(y=1)$  (Shifted up by 1)
- point on graph 1 unit right of the vertical asymptote:  $(4,3)$   
(Vertical distance is  $2 \cdot 1 = 2$ )
- point on graph 2 units right of the vertical asymptote  $(5,2)$   
(Vertical distance is  $2 \cdot 1/2 = 1$ )
- point on graph  $1/2$  unit right of the vertical asymptote:  
 $(3\frac{1}{2}, 5)$  (Vertical distance is  $2 \cdot 2 = 4$ )
- Points on lower left curve:  $(1,0)$ ,  $(2,-1)$ ,  $(2\frac{1}{2}, -3)$



Example 8: Rational Function  $y = \frac{-2x+3}{x+1}$

Points:

- Vertical asymptote  $x=-1$
- Horizontal asymptote  $y=-2$
- Points:

x	y
0	3
1	$\frac{1}{2}$
2	$-\frac{1}{3}$

- Graphed points on lower left curve by using point ins table:  $(0,3)$  is 1 unit to the right and 5 units up from where the asymptotes meet; the corresponding point is ) is 1 unit to the left and 5 units down, at  $(-2,-7)$
- 
- (Note: if a point on the first curve is  $x$ -units right and  $y$ -units up from the asymptotes, then the corresponding point on the other curve will be  $x$ -units left and  $y$ -units down)

