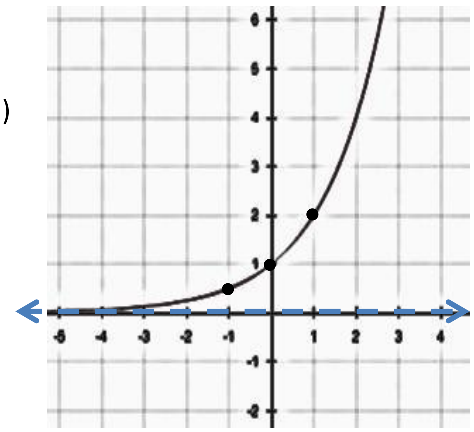
Graphing with Points – Exponential and Logarithmic Functions

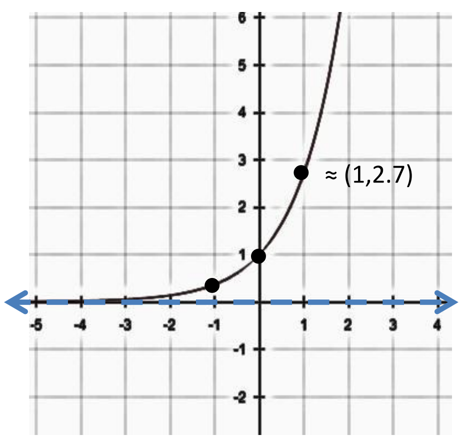
This is a supplement to the earlier "graphing with points" handout that covered lines, quadratics, cubics, absolute value, and reciprocal functions.

In the text book (Sec 3.1 and 3.2), when graphing of exponential and logarithmic functions is introduced, tables (with approx. 7 x-values) are used. The book then discusses how to graph these types of graphs using transformations. Our approach here will be to use three key points (and the asymptote!) and determine where these points go using transformations.



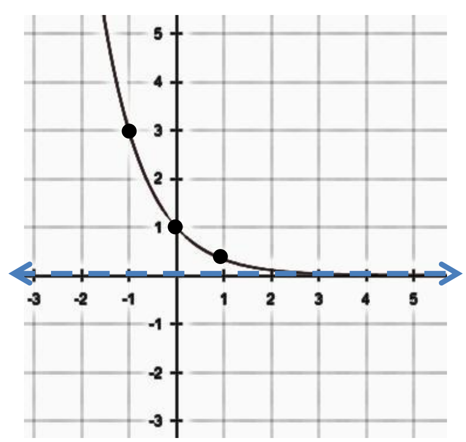
Exponential Functions (general form $y = a^x$, at right $y = 2^x$)

- HORIZONTAL asymptote
  - in general and picture shown , y=0
- Points with x-values: x=-1, x=0, x=1
  - these are three consecutive x-values
  - in general (-1 , 1/a), (0,1), (1,a)
  - shown (-1 , ½), (0,1), (1,2).



Example 1: natural base "e": $y = e^x$

- Memorize that e≈2.7
- HORIZONTAL asymptote: y=0
- Points: (-1 , 1/e), (0,1), (1,e)

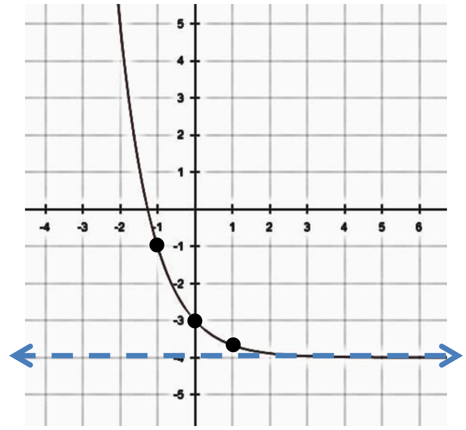


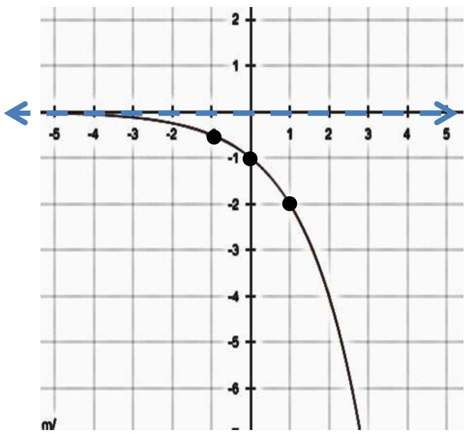
Example 2: value of *a* less than 1 (equivalent to negative exponent) :
$y = (\frac{1}{3})^x = 3^{-x}$

- (reflection in y-axis)
- HORIZONTAL asymptote: y=0
- Points: (-1 , 3), (0,1), (1,1/3)

Example 3: vertical shift : $y = (\frac{1}{3})^x - 4 \ = 3^{-x} - 4$

- HORIZONTAL asymptote: y= -4
- Points: (-1 , -1), (0,-3), $(1, -3\frac{2}{3})$





Example 4: negative: $y = -2^{\ x}$

- (reflection in x-axis)
- HORIZONTAL asymptote: y=0
- Points: (-1 , -½), (0,-1), (1,-2)
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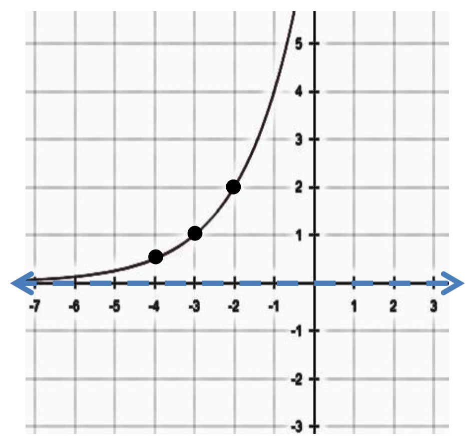
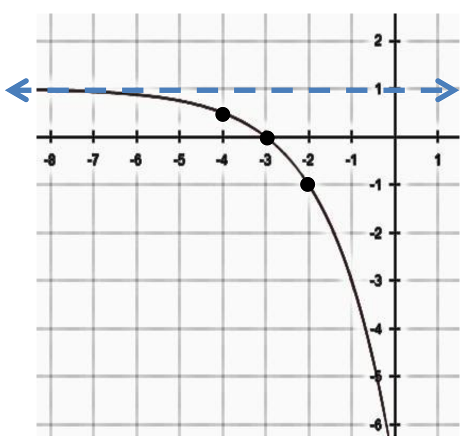
Example 5: horizontal shift: $y = 2^{(x+3)}$

- HORIZONTAL asymptote: y=0
- Points: (-4 , ½), (-3,1), (-2,2)

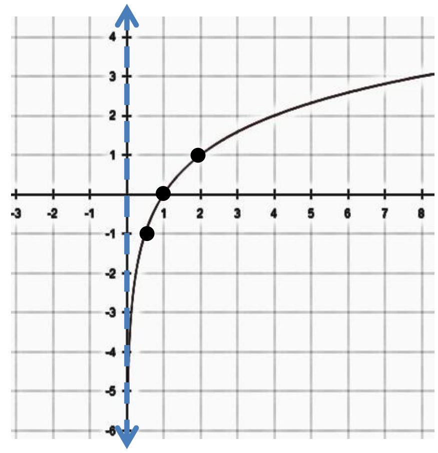




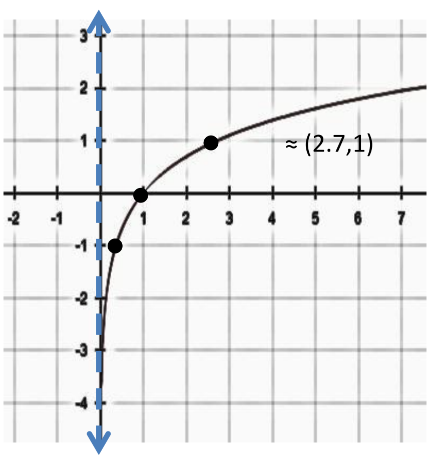
Example 6: multiple transformations: $y = -2^{(x+3)} + 1$

- HORIZONTAL asymptote: y=1
- Points: (-1 , 1/2), (0,1), (1,2) → (-1 , -1/2), (0,-1), (1,-2) (reflection) → (-4 , -1/2), (-3,-1), (-2,-2) (horizontal shift) → (-4 , ½), (-3,0), (-2,-1) (vertical shift)

Logarithmic Functions (general form $\log_a x$, at right $\log_2 x$)

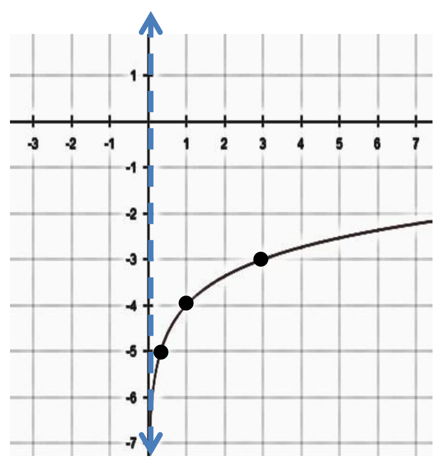


- VERTICAL asymptote
  - in general and picture shown , x=0
- Points : - note the x-values are not consecutive; they depends on a
  However the y-values are consecutive
  - in general (1/a , -1), (1,0), (a,1)
  - shown (1/2 , -1), (1,0), (2,1)



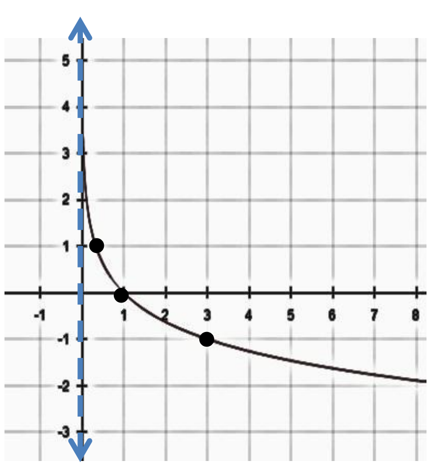
Example 7: natural log "e": $\log_e x = \ln x$

- Memorize that e≈2.7
- VERTICAL asymptote: x=0
- Points : (1/e , -1), (1,0), (e,1)



Example 8: vertical shift : $\log_3 x - 4$

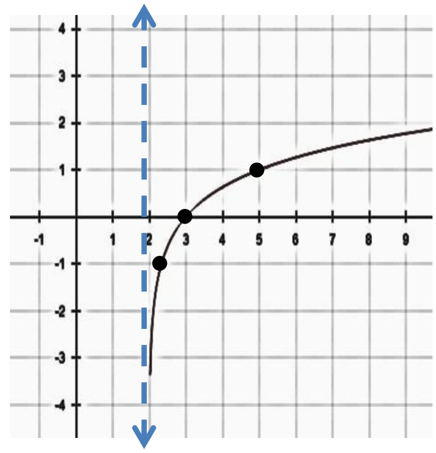
- VERTICAL asymptote: x=0
- Points : (1/3 , -5), (1,4), (3,3)



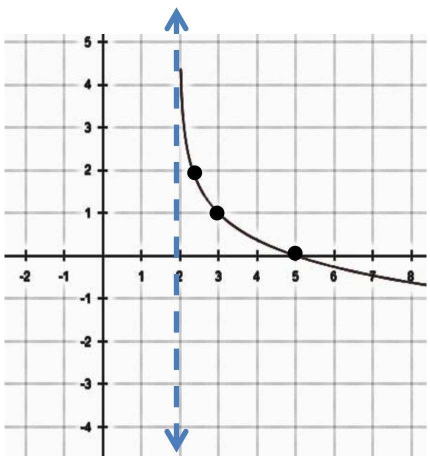
Example 9: negative : $-\log_3 x$

- Reflection in x-axis
- VERTICAL asymptote: x=0
- Points : (1/3 , 1), (1,0), (3,-1)

Example 10: horizontal shift : $\log_3(x-2)$



- VERTICAL asymptote: x=2
- Points : ($2\frac{1}{3}$, -1), (3,0), (5,1)



Example 11: multiple transformations : $-\log_3(x-2)+1$

- VERTICAL asymptote: x=2
- Points : (1/3 , -1), (1,0), (3,1) →(1/3 , 1), (1,0), (3,-1) (reflection)

  →($2\frac{1}{3}$, 1), (3,0), (5,-1) (horizontal shift)

  →($2\frac{1}{3}$, 2), (3,1), (5,0) (horizontal shift)