Math 1050 Review

#12 on Exam 2

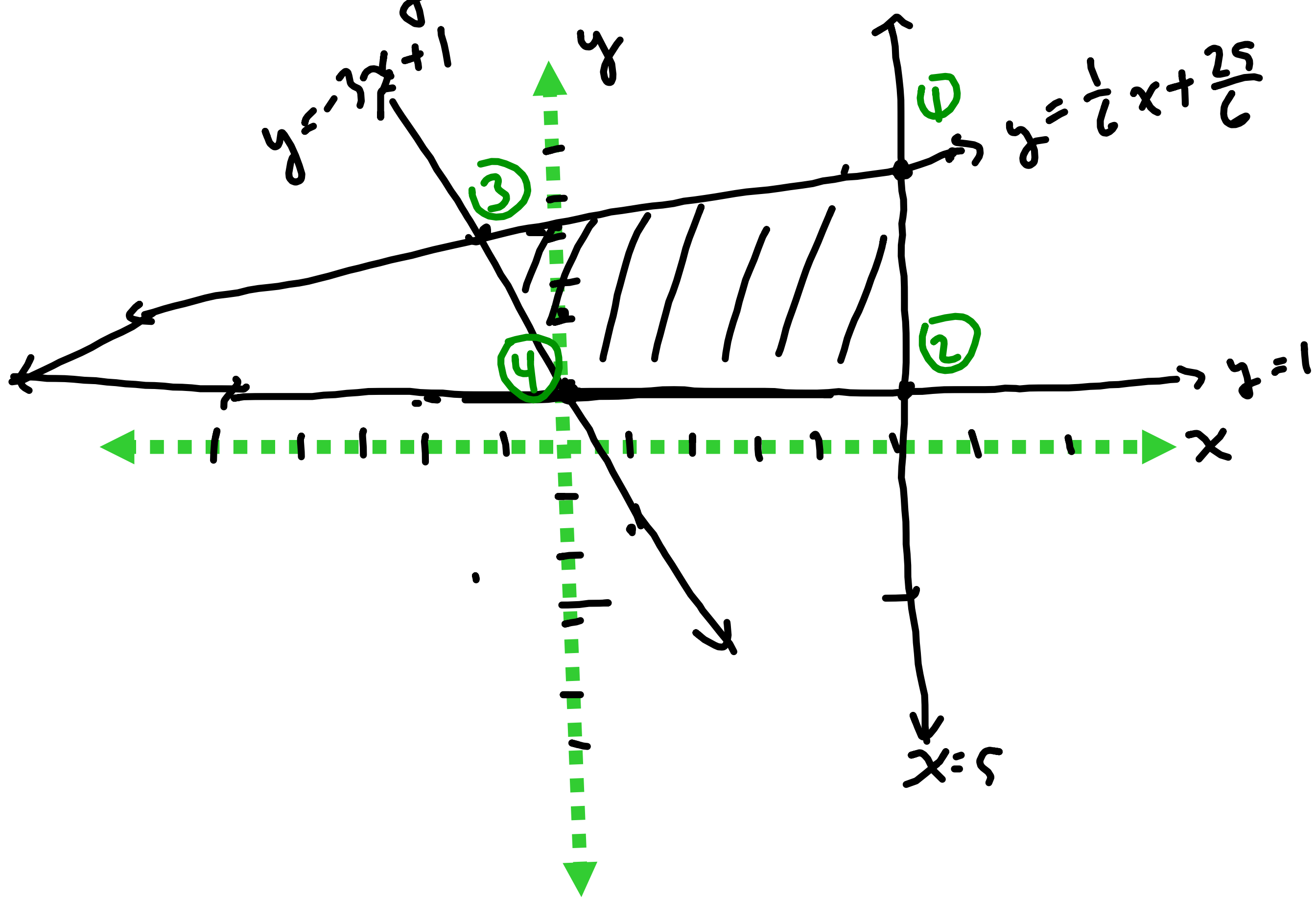
$x \le 5$

$y \ge 1$

$6y \le x + 25 \Rightarrow y \le \frac{1}{6}x + \frac{25}{6}$

$y - 1 \ge -3x \Rightarrow y \ge -3x + 1$

(a) Graph & shade the region determined by the constraints.



(b) List vertex corner points

(1) $y = \frac{1}{6}x + \frac{25}{6}$ & $x = 5$

$\Rightarrow y = \frac{5}{6} + \frac{25}{6} = \frac{30}{6} = 5$

$(5,5)$

(2) $x = 5, \; y = 1$

$(5,1)$

(3) $y = -3x + 1, \; y = \frac{1}{6}x + \frac{25}{6}$

$\Rightarrow -3x + 1 = \frac{1}{6}x + \frac{25}{6}$

$\Rightarrow -3x - \frac{1}{6}x = \frac{25}{6} - 1$

$\Rightarrow -19/6\, x = 19/6$

$\Rightarrow x = -1$

so $y = -3(-1) + 1$
$= 4$

$(-1,4)$

(4) $y = 1, \; y = -3x + 1$

$\Rightarrow 1 = -3x + 1$

$\Rightarrow 0 = -3x$

$\Rightarrow x = 0$

$(0,1)$

(c) For $z = 2x - 3y$ find max z-value & point where it occurs.

Plug in the four vertices

① $(5,5)$
$z = 2(5) - 3(5)$
$\Rightarrow z = -5$

② $(5,1)$
$z = 2(5) - 3(1)$
$= 7$

③ $(-1,4)$
$z = 2(-1) - 3(4)$
$= -14$

④ $(0,1)$
$z = 2(0) - 3(1)$
$= -3$

So max value is 7
at $(5,1)$.

#11 on Exam 2

Write partial fraction decomposition

for $\dfrac{5x+13}{(x+3)^2} = \dfrac{A}{(x+3)} + \dfrac{B}{(x+3)^2}$

multiply both sides by $(x+3)^2$:

$$5x+13 = A(x+3) + B$$

pick $x=-3$:

$$-15+13 = A(0) + B \implies -2 = B$$

or $B=-2$.

set $x=0$:

$$0+13 = A(0+3) + (-2)$$

$$\implies 13 = 3A - 2 \implies 15 = 3A$$

$$A = 5.$$

So $\dfrac{5x+13}{(x+3)^2} = \dfrac{5}{x+3} + \dfrac{-2}{(x+3)^2}$.

Ex: Find the partial fraction decomp.

for $\dfrac{2x+1}{x^2(x^2+x+1)}$

irreducible

form: $$\frac{2x+1}{x^2(x^2+x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+x+1}$$

$$\Rightarrow 2x+1 = Ax(x^2+x+1) + B(x^2+x+1) + (Cx+D)x^2$$

set $x=0$: $1 = A(0)(0+0+1) + B(0+0+1) + (0+D)(0)$

$\Rightarrow \underline{1 = B}$ pick other x-values

set $x=1$: $3 = 3A + 3 + C + D$ (1)

$\Rightarrow 0 = 3A + C + D$

set $x=-1$: $-1 = -A + 1 + -C + D$ (2)

$\Rightarrow -2 = -A - C + D$

Set $x=2$:

$$5 = 14A + 7 + 8C + 4D$$

$$\Rightarrow -2 = 14A + 8C + 4D$$

$$\Rightarrow -1 = 7A + 4C + 2D \quad (3)$$

$$(1) + (2) \Rightarrow -2 = 2A + 2D \Rightarrow -1 = A + D$$

$$4(2) + (3) \Rightarrow -9 = 3A + 6D \Rightarrow -3 = A + 2D$$

$$\Rightarrow 2 = -D \Rightarrow D = -2$$

$\Rightarrow A = 1$ Plug into (1)

$$0 = 3 + C - 2 \Rightarrow 0 = 1 + C$$

$$\Rightarrow C = -1.$$

Finally:

$$\frac{2x+1}{x^2(x^2+x+1)} = \frac{1}{x} + \frac{1}{x^2} + \frac{-x-2}{x^2+x+1}$$

4(b) on Exam 2

$\frac{3}{2} + \log_4 x^2 = 2$

$\Rightarrow \log_4 x^2 = 1/2$

$\Rightarrow 4^{1/2} = x^2$

$\Rightarrow 2 = x^2$

$\Rightarrow x = \pm\sqrt{2}$

check:

both ok.

first isolate log

side:

$\log_b a = c$

is equivalent to

$b^c = a$

#11 on W.A. 9.2 pt. 1

Find the first 5 terms of arithmetic sequence with $a_4 = 37$, $a_{10} = 91$

$a_n = a_1 + (n-1)d$

$\Rightarrow a_4 = 37 = a_1 + 3d$

$\Rightarrow a_{10} = 91 = a_1 + 9d$

2 equations & 2 unknowns

$\Rightarrow 91 = a_1 + 3d + 6d$

$\Rightarrow 91 = 37 + 6d \quad \Rightarrow 54 = 6d$

$\Rightarrow d = 9 \quad \Rightarrow 37 = a_1 + 27$

$\Rightarrow a_1 = 10, \; a_2 = 19, \; a_3 = 28,$

$a_4 = 37, \ldots$

#10 on 9.5 pt. 2

Use binomial thrm to expand

$(7-\sqrt{5}\,i)^4$

binomial: $(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$

$= \sum_{0}^{4} \binom{4}{r} 7^{4-r} (-\sqrt{5}\,i)^r$

$= 1\cdot 7^4 (-\sqrt{5}i)^0 + 4\cdot 7^3 (-\sqrt{5}\,i)^1$

$+ 6\cdot 7^2 (-\sqrt{5}i)^2 + 4\cdot 7^1 (-\sqrt{5}i)^3$

$+ 1\cdot 7^0 (-\sqrt{5}i)^4$

$$\begin{array}{ccccccccc} &&&& 1 &&&& \\ &&& 1 && 1 &&& \\ && 1 && 2 && 1 && \\ & 1 && 3 && 3 && 1 & \\ 1 && 4 && 6 && 4 && 1 \end{array}$$

$= 7^4 - \sqrt{5}\cdot 4\cdot 7^3 i + 6\cdot 7^2(-5) + 28\cdot 5\sqrt{5}\,i$

$+25$

$= 2401 - 1372\sqrt{5}\,i - 1470 + 140\sqrt{5}\,i + 25$

$= 956 - 1232\sqrt{5}\,i$