Math1210 Midterm 3 Extra Review Key

1. Evaluate

(a) $\int \left(2x^4(x^5-1)^{-2/3}\right)dx=\frac{6}{5}(x^5-1)^{1/3}+C$

(b) $\int \left(3\sqrt[5]{t}-\frac{4}{t^2}+2t^3-\sin t+10\right)dt=\frac{5}{2}t^{6/5}+\frac{4}{t}+\frac{1}{2}t^4+\cos t+10t+C$

(c) $\int \frac{(2x+3)^2}{\sqrt{x}}dx=\frac{8}{5}x^{5/2}+8x^{3/2}+18\sqrt{x}+C$

(d) $\int (4x^5-\cos x+\sqrt[3]{x^2})dx=\frac{2}{3}x^6-\sin x+\frac{3}{5}x^{5/3}+C$

(e) $\int \frac{4x}{\sqrt{x^2-3}}dx=4\sqrt{x^2-3}+C$

(f) $\int (2x^3\sqrt{2x^4+3})dx=\frac{1}{6}(2x^4+3)^{3/2}+C$

2. Solve the following differential equation.

$$\frac{dy}{dx}=\frac{4x^3+\frac{1}{x^2}}{3y^4} \quad \text{such that} \quad y=-1 \quad \text{when} \quad x=1$$

Answer: $y=\sqrt[5]{\frac{5}{3}x^4-\frac{5}{3x}-1}$

3. For the function $f(x)=\frac{3x-2}{x-5}$ on the closed interval [1, 4], decide whether or not the Mean Value Theorem for Derivatives applies. If it does, find all possible values of c. If not, then state the reason.

Answer: Yes MVT applies because the function is continuous and differentiable on [1, 4].
c = 3

4. Solve $x^4-53=0$ using Newton's Method, accurate to four decimal places.

Use $x_{n+1}=x_n-\frac{x_n^4-53}{4x_n^3}=\frac{4x_n^4-x_n^4+53}{4x_n^3}=\frac{3x_n^4+53}{4x_n^3}$. If you start with $x_1=2.5$ (why? Because I know that $2^4=16$ and $3^4=81$ and 53 is somewhere between 16 and 81), then you'll get these values out: 2.5, 2.723, 2.698505497, 2.69816794, and 2.698167876. So the answer is approximately 2.6982 to four decimal places.

5. For $f(x)=3x^2+4x-1$ on [0, 2], decide whether or not the Mean Value Theorem (for Derivatives) applies. If it does, find all possible values of c. If not, then state the reason.

Answer: Yes MVT applies because the function is continuous and differentiable everywhere.
c = 1

6. Solve this equation using (A) the Bisection Method **and** (B) Newton's Method to three decimal places.

$$f(x)=2x^3-4x+1=0 \quad \text{On } [0, 1]$$

Answer: should get (A) the midpoint of the interval from 0.2578125 to 0.26171875 which would be 0.259765625 which is about 0.2598 and (B) 0.2586

7. Solve this differential equation. $\frac{dy}{dx} = \frac{x+3x^2}{y^2}$ and $y=2$ when $x=0$

Answer: $y = \sqrt[3]{\frac{3}{2}x^2 + 3x^3 + 8}$

8. Evaluate $\sum_{i=1}^{10} [(i-2)(2i+5)] = 725$

9. Evaluate the definite integral **using the definition** (the tedious way).

$\int_{-1}^{2} (5x-1)\,dx$. (Note: Here is the definition. $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$)

Answer: $\Delta x = \frac{3}{n}$, $x_i = -1 + \frac{3i}{n}$, $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left(\frac{-18}{n} + \frac{45i}{n^2} \right)$, $\int_{-1}^{2} (5x-1)\,dx = 4.5$

10. Evaluate $\sum_{i=1}^{10} [(3i-4)(i+5)] = 1640$

11. Evaluate the definite integral **using the definition** (the tedious way). $\int_0^3 (4x^2-1)\,dx$.

Answer: $\Delta x = \frac{3}{n}$, $x_i = \frac{3i}{n}$, $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left(\frac{108i^2}{n^3} - \frac{3}{n} \right)$, $\int_0^3 (4x^2-1)\,dx = 33$