Math1220 Midterm 1 Review Problems
Answer Key

1. $y=-4x+4+\frac{\pi}{2}$

2. $f^{-1}(x)=\frac{1+5\sqrt[3]{x}}{2-2\sqrt[3]{x}}$

3.

(a) $y'=\frac{2\cos(3x)(-\sin(3x))(3)}{\cos^2(3x)}+\frac{3}{\sqrt{1-(3x-2)^2}}$

(b) $y'=(5x+3)^{2x^2}\left(4x\ln(5x+3)+\frac{5(2x^2)}{5x+3}\right)$

(c) $y'=\pi(1+x^4)^{\pi-1}(4x^3)+\pi^{1+x^4}(\ln\pi)(4x^3)$

(d) $y'=-sech(\cos(2x))\tanh(\cos(2x))(-\sin(2x))(2)$

(e) $y'=\frac{3}{3x-2}-12x^{-7}+12x^2-5\cos(5x)$

(f) $y'=e^{\frac{1}{3x}}\left(\frac{-1}{3x^2}\right)+\frac{1}{e^{3x}}(-3)$

(g) $y'=(x^3-1)^{\ln x}\left(\frac{1}{x}\ln(x^3-1)+\frac{3x^2(\ln x)}{x^3-1}\right)$

(h) $y'=\frac{-\sin x}{\sqrt{(\cos x+3)^2-1}}$

4. $t=\frac{-10\ln(0.08)}{\ln 2}$ years

5. Evaluate each integral.

(a) $x\arcsin(2x)+\frac{1}{2}\sqrt{1-4x^2}+C$

(b) $5\ln|2x^2+x-7|+C$

(c) $-5\arctan(\ln x)+C$

(d) $\frac{4^{-1}-4^{-5}}{\ln 16}=\frac{255}{1024\ln 16}$

(e) $\frac{1}{2}\left(y^2\arctan(y)-y+\arctan(y)\right)+C$

(f) $\frac{-1}{\ln 2}\left(2^{\sqrt{3}/2}-2\right)$

(g) $\frac{-1}{3}\ln 4$

(h) $\frac{6^4-1}{\ln 36}$

(i) $\frac{1}{2}\ln(e^{2x}+5)+C$

(j) $\frac{5}{3}(\arcsin(x^3))+C$

(k) $\frac{1}{4}\arctan\left(\frac{x^2}{2}\right)+C$

(l) $\frac{1}{4}\ln(x^4+4)+C$

(m) $\frac{3}{\ln 4}\left(x(4^x)-\frac{4^x}{\ln 4}\right)$

(n) $-\frac{\pi}{2}$

6. $\frac{1}{14}$

7. $f'(x)=\frac{\sin x+1}{\cos^2 x}$ and since $\sin x$ is always between -1 and 1, then $1+\sin x$ must be between 0 and 2 (inclusive) which is always nonnegative. The denominator is also always positive on the interval $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$. This means that the derivative is always nonnegative in the given domain of the function. This implies that the function is monotonically increasing.

8. $\frac{1}{4}$

9. $f'(x)=-\left(\frac{2}{1+4\text{x}^2}+15(x-1)^2\right)$ which is always positive inside the parentheses since all coefficients are positive and the powers on x are even. Thus, the derivative is always negative which means the inverse function exists.

$(f^{-1})'(11)=\frac{1}{f'(0)}=\frac{-1}{17}$

10.

(a) $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{5\text{x}}=e^{15}$

(b) $\lim_{x\to\infty}(1)^{5\text{x}}=1$