Math2210 Midterm 2 (11.9, 12.1-12.8) Spring, 2015 Instructor: Kelly MacArthur

uid number: Key

Special number: __________

Instructions:

- Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown.
- All answers should be completely simplified, unless otherwise stated.
- There are no calculators or any sort of electronics allowed on this exam. Make sure all cell phones are put away and out of sight. If you have a cell phone out at any point, for any reason, you will receive a zero on this exam.
- You will be given an opportunity to ask clarifying questions about the instructions at exactly 8:40 a.m. (for a couple minutes). The questions will be answered for the entire class. After that, no further questions will be allowed, for any reason.
- You must show us your U of U student ID card when finished with the exam.
- The exam key will be posted on Canvas by noon.
- Somewhere on this exam, you may need to know
  - $1/8 = 0.125$
  - $D_x(\arctan x) = \dfrac{1}{1+x^2}$
- You may ask for scratch paper. You may use NO other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch page.
- You are allowed to use one 8.5x11 inch piece of paper with notes for your reference during the exam.

(This exam totals 100 points, not including the extra credit problem.)

STUDENT—PLEASE DO NOT WRITE BELOW THIS LINE. THIS TABLE IS TO BE USED FOR GRADING.

Problem	Score
1 & 2	
3	
4 & 5	
6 & 7	
8	
9	
EC	

Total Percentage:	

1. (12 pts) Find the directional derivative of $f(x,y,z)=x^2y^2+yz^3-4xyz$ at $p=(3,0,1)$ in the direction of $a=i-2j+2k$.

$\nabla f = \langle 2xy^2-4yz,\ 2x^2y+z^3-4xz,\ 3yz^2-4xy \rangle$

$\nabla f(3,0,1) = \langle 0-0,\ 0+1-4(3),\ 0-0 \rangle = \langle 0,-11,0 \rangle$

$\hat{a} = \dfrac{\langle 1,-2,2 \rangle}{\sqrt{1+4+4}} = \langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3} \rangle$

$\hat{a} \cdot \nabla f(3,0,1) = \frac{1}{3}(0) + \frac{-2}{3}(-11) + \frac{2}{3}(0) = \frac{22}{3}$

Answer: $22/3$

2. (10 pts) Use the total differential dz to approximate the change in z as (x, y) moves from P(-4, 0.25) to Q(-4.01, 0.26) for $z=\arctan(xy)$.

$dx = -4.01-(-4) = -0.01$

$dy = 0.26-0.25 = 0.01$

$dz = \nabla f(x,y) \cdot \langle dx, dy \rangle$

$\nabla f = \langle \dfrac{y}{1+x^2y^2},\ \dfrac{x}{1+x^2y^2} \rangle$

choose $x=-4, y=0.25=\frac{1}{4}$

$\nabla f(-4,\frac{1}{4}) = \langle \dfrac{1/4}{1+4^2(\frac{1}{4^2})},\ \dfrac{-4}{1+4^2(\frac{1}{4^2})} \rangle = \langle \frac{1}{8}, \frac{-4}{2} \rangle = \langle \frac{1}{8}, -2 \rangle$

$dz = \langle \frac{1}{8}, -2 \rangle \cdot \langle -0.01, 0.01 \rangle = \langle 0.125, -2 \rangle \cdot \langle -0.01, 0.01 \rangle$

$= -.00125 - 0.02 = -0.02125$

Answer: -0.02125

3. (10 pts) Find the two points on the 3-d surface $F(x,y,z)=x^2+2y^2-z^2-7=0$ where the tangent plane is parallel to the plane $3x+2y-2z=15$.

$\nabla F = \langle 2x, 4y, -2z\rangle = \langle 3, 2, -2\rangle k$ $\quad k \in \mathbb{R}$

$2x = 3k$ $\quad 4y = 2k$ $\quad -2z = -2k$

$x = \frac{3}{2}k$ $\quad y = \frac{k}{2}$ $\quad z = k$

$\Rightarrow$ plug into F:

$$\left(\frac{3}{2}k\right)^2 + 2\left(\frac{k}{2}\right)^2 - k^2 = 7$$

$$4\left(k^2\left[\frac{9}{4} + \frac{1}{2} - 1\right]\right) = (7)4$$

$$k^2\left[9 + 2 - 4\right] = 28$$

$$k^2(7) = 28$$

$$k^2 = 4$$

$$k = \pm 2$$

$k = 2$
$\Rightarrow x = 3,\ y = 1,\ z = 2$

$k = -2$
$\Rightarrow x = -3,\ y = -1,\ z = -2$

Answer: $(3,1,2)$ and $(-3,-1,-2)$

4. (10 pts) Describe the largest set S (*algebraically and geometrically—for your geometric description, just describe/name the surface where it's continuous*) on which $f(x,y,z)=\dfrac{3xy}{\ln(25-x^2-y^2-z^2)}$ is continuous.

we need $25-x^2-y^2-z^2>0 \iff x^2+y^2+z^2<25$
(inside sphere of radius 5)

But we also need $25-x^2-y^2-z^2\neq 1$
$\iff x^2+y^2+z^2\neq 24$ (not on sphere of radius $\sqrt{24}$)

Answer: $\{(x,y,z): x^2+y^2+z^2<25$ and $x^2+y^2+z^2\neq 24\}$

5. (15 pts) For $e^{xy}+x\sin y+2z^2=9$, find the equation of the tangent plane at the input point (2, 0), in the first octant.

$F(x,y,z)=e^{xy}+x\sin y+2z^2-9=0$

$\nabla F(x,y,z)=\langle ye^{xy}+\sin y,\ xe^{xy}+x\cos y,\ 4z\rangle$

full point: $(2,0,2)$

$e^{2(0)}+2\sin(0)+2z^2=9$

$1+2z^2=9$

$2z^2=8$, $z^2=4$, $z=\pm 2$

but we want Octant I pt

$\Rightarrow z=2$

$\nabla F(2,0,2)=\langle 0+0,\ 2e^0+2\cos 0,\ 4(2)\rangle$

$=\langle 0,4,8\rangle=4\langle 0,1,2\rangle$

tangent plane: $\langle 0,1,2\rangle\cdot\langle x-2,y,z-2\rangle=0$

$y+2z-4=0$

$y+2z=4$

Answer: $y+2z=4$

6. (10 pts) Given $w(x,y)=2x^3-xy+2y^2$, $x^3=u-v$, $y=uv$, answer these questions.

(a) Write out the chain rule for $\frac{\partial w}{\partial u}$ that you need to use for this problem.

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u}$$

(b) Find $\frac{\partial w}{\partial u}$ when $u=-3$, $v=-2$.

$$\frac{\partial (x^3)}{\partial u} = \frac{\partial (u-v)}{\partial u}$$

$$3x^2 \frac{\partial x}{\partial u} = 1$$

$$\frac{\partial x}{\partial u} = \frac{1}{3x^2}$$

$$\frac{\partial y}{\partial u} = v$$

when $u=-3, v=-2$:

$x^3 = -3-(-2) = -1 \Rightarrow x=-1$

$y = -3(-2) = 6$

$$\frac{\partial w}{\partial u} = (6x^2-y)\left(\frac{1}{3x^2}\right) + (-x+4y)(v)$$

$$\left.\frac{\partial w}{\partial u}\right|_{u=-3, v=-2} = (6(-1)^2-6)\left(\frac{1}{3(-1)^2}\right) + (-(-1)+4(6))(-2) = 0 + 25(-2) = -50$$

Answer : -50

7. (10 pts) Find the limit, if it exists. (Show all your reasoning.)

$$\lim_{(x,y)\to(0,0)} \frac{3x-4y}{2x+y}$$

along $x=0$: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x-4y}{2x+y} = \lim_{y\to 0} \frac{-4y}{y} = -4$

along $y=0$: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x-4y}{2x+y} = \lim_{x\to 0} \frac{3x}{2x} = \frac{3}{2}$

$-4 \neq \frac{3}{2} \Rightarrow$ limit DNE

Answer : DNE

8. (a) (5 pts) Convert $4x^2=25y-4y^2$ from a Cartesian coordinate equation into an equation in cylindrical coordinates.

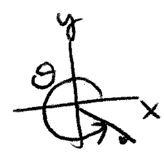
$$4x^2+4y^2=25y$$
$$4(x^2+y^2)=25y$$
$$4r^2=25r\sin\theta$$
$$4r=25\sin\theta$$
$$r=\frac{25}{4}\sin\theta$$

Answer: $r=\frac{25}{4}\sin\theta$

(b) (5 pts) Convert $(2\sqrt{2},-2\sqrt{2},4\sqrt{3})$ from Cartesian coordinates into spherical coordinates.
$x \quad y \quad z$

$$\rho=\sqrt{x^2+y^2+z^2}=\sqrt{8+8+48}=\sqrt{64}=8$$

$$\tan\theta=\frac{y}{x} \Rightarrow \tan\theta=\frac{-2\sqrt{2}}{2\sqrt{2}}=-1$$



$$\Rightarrow \theta=\frac{7\pi}{4}$$

$$\frac{z}{\rho}=\cos\varphi \Rightarrow \cos\varphi=\frac{4\sqrt{3}}{8}=\frac{\sqrt{3}}{2}$$

$$\varphi=\frac{\pi}{6}$$

Answer: $\left(8, \frac{7\pi}{4}, \frac{\pi}{6}\right)$

9. (13 pts) Find all critical and saddle points for $f(x,y)=x^3+y^2-3x^2+2y+4$.
Determine whether each point is a minimum, maximum or saddle point.

$f_x = 3x^2-6x = 0$

$3x(x-2)=0$

$x=0$ or $x=2$

$f_y = 2y+2=0$

$y=-1$

$f_{xx} = 6x-6$

$f_{yy}=2$

$f_{xy}=0$

$\Rightarrow D= 12x-12$

input pts to consider: ① $(0,-1)$
② $(2,-1)$

① $D=12(0)-12<0 \Rightarrow (0,-1)$ is saddle

② $D=12(2)-12=12>0 \Rightarrow (2,-1)$ is local min pt.
and $f_{xx}=6(2)-6>0$

① $z=f(0,-1)=0^3+(-1)^2-3(0^2)+2(-1)+4 = 1-2+4=3$

② $z=f(2,-1)= 2^3+(-1)^2-3(2^2)+2(-1)+4$
$=8+1-12-2+4 = 13-14=-1$

Critical point(s) (Specify whether they're min, max or saddle.):
Note: Give FULL 3-d points.

$(0,-1,3)$ saddle

$(2,-1,-1)$ min

Extra Credit: (5 pts) Answer each of these short answer questions.

1. Circle all of the following statements that are true. (It's possible more than one of these statements is correct.)

(a) The vector $< f_x(x,y)\ ,\ f_y(x,y)\ ,\ -1 >$ is perpendicular to the surface $z=f(x,y)$ at the point *(x, y)*.

(b) The vector $< f_x(x,y)\ ,\ f_y(x,y) >$ is perpendicular to the surface $z=f(x,y)$ at the point *(x, y)*.

(c) The vector $< f_x(x,y)\ ,\ f_y(x,y) >$ is perpendicular to the level curve of the surface $z=f(x,y)$ at the point *(x, y)*.

2. Which of the following two statements is true? (circle one)

(a) If f is continuous at (x, y), then it's differentiable there.

(b) If f is differentiable at (x, y), then it's continuous there.

3. If $\lim_{y\to 0} f(y,y)=L$, then $\lim_{(x,y)\to(0,0)} f(x,y)=L$.

True or False (circle one)

4. The level curves for $z=\sqrt{25-2x^2-y^2}$ are circles.

True or False (circle one)

If your answer is false, then state what shape the level curves are:

ellipses

$z^2=25-2x^2-y^2$

$2x^2+y^2+z^2=25$