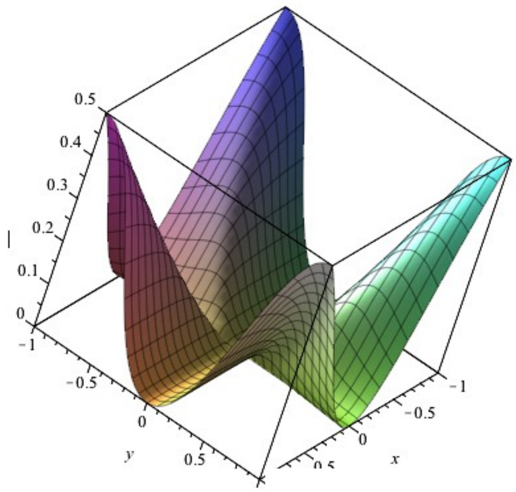
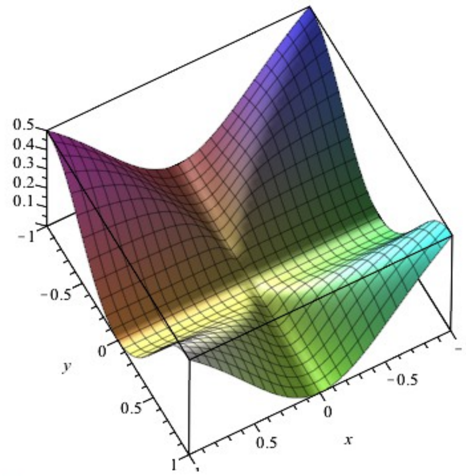
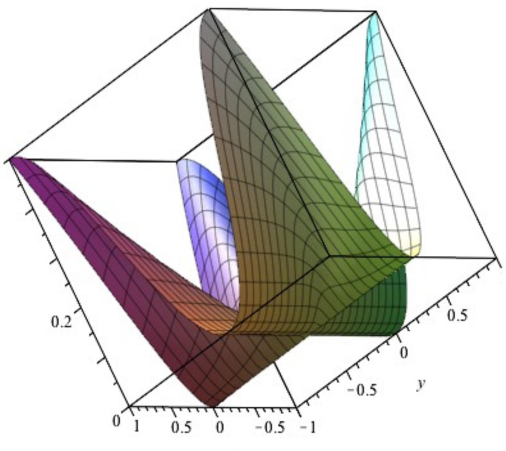
Graph of surface $z = \dfrac{x^2 y^2}{x^2 + y^4}$ (from several perspectives). You can see from the generated graph that it goes through the origin, so the point is well-defined there.







So, we already had $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y^2}{x^2+y^4} = \lim\limits_{r\to 0} \dfrac{r^4(\cos^2\theta \sin^2\theta)}{r^2(\cos^2\theta + r^2\sin^2\theta)} = \lim\limits_{r\to 0} \dfrac{r^2(\cos^2\theta\sin^2\theta)}{\cos^2\theta + r^2\sin^2\theta} = 0$ as long as $\cos\theta \neq 0$.

And, if $\cos\theta = 0$, that's equivalent to $\theta = \dfrac{(2n+1)\pi}{2}$, $n \in \mathbb{Z}$. So, we can figure out what happens as $\theta \to \dfrac{(2n+1)\pi}{2}$.

$$\lim_{\theta\to\frac{(2n+1)\pi}{2}} \frac{r^2(\cos^2\theta\sin^2\theta)}{\cos^2\theta + r^2\sin^2\theta}$$ is the 0/0 case, so we can use L'Hopital's Rule to get

$$= \lim_{\theta\to\frac{(2n+1)\pi}{2}} \frac{-2r^2\sin^2\theta + 2r^2\cos^2\theta}{-2+4r^2\sin^2\theta} \quad \text{(after simplifying)} \quad = \frac{-2r^2}{-2+4r^2} = \frac{r^2}{1-2r^2}$$ and as r goes to zero here, we still get 0. This completes the proof that the limit exists and goes to 0.