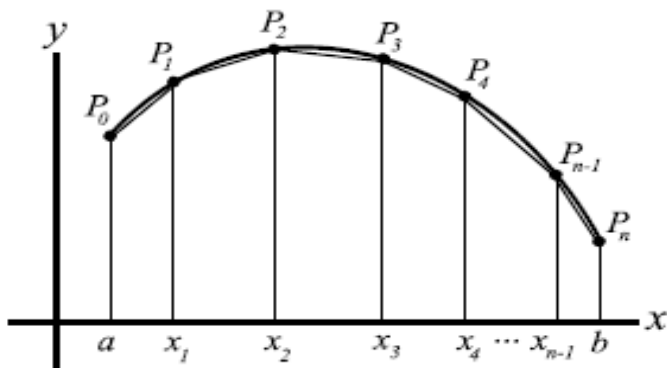


Math6100
Day 10 Notes

7.1 & 7.2, Lengths of Curves in the Plane & Areas of Surfaces of Revolution

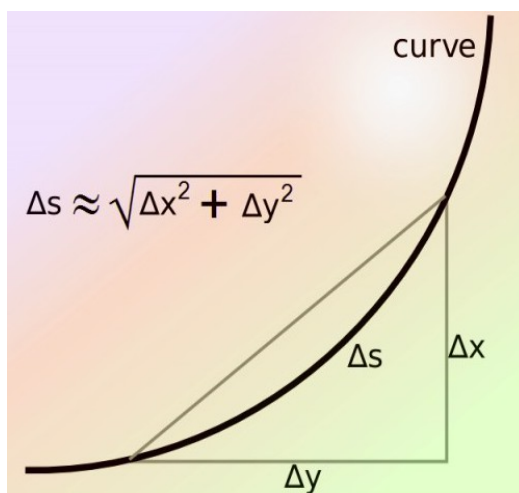
7.1 Lengths of Curves in the Plane

How can we approximate the length of a given curve, $y=f(x)$, on the interval $[a, b]$?



We can say that the length of the curve is approximately equal to the sum of the lengths of each of the line segments connecting discrete points along the curve.

Let's zoom in and see if we can formalize this into a formula.



$\Delta x_i = x_{i+1} - x_i$, $\Delta y_i = y_{i+1} - y_i = f(x_{i+1}) - f(x_i)$ But remember that

$f'(x_{i+1}) \approx \frac{f(x_{i+1}) - f(x_i)}{\Delta x_i}$. This means $\Delta y_i \approx f'(x_{i+1}) \Delta x_i$ so

$$\Delta s \approx \sqrt{(\Delta x_i)^2 + (f'(x_{i+1}) \Delta x_i)^2} = \sqrt{1 + (f'(x_{i+1}))^2} \Delta x_i .$$

So, the approximate arc length is:

What is the exact arc length formula then?

Ex 1: Find the length of the curve $y=x^2$ between $(0,0)$ and $(2,4)$.

Ex 2: A cable hanging between two poles of the same height usually takes the shape of a catenary. What is the length of the cable that is hanging between two poles 40 m apart if this cable takes the shape of the catenary $y = 10 \left(e^{\frac{x}{20}} + e^{\frac{-x}{20}} \right)$?

Ex 3: Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ for $x \in [0, 2]$.

Ex 4: Can we use this formula (or another parametric version of it) to help us find the circumference of a circle?

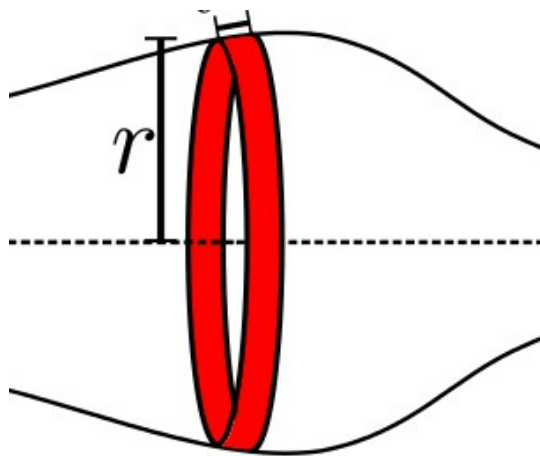
Ex 5: Find the arc length of the curve given by

$$x = \frac{1}{4}t^4 + 5 \quad \text{and} \quad y = \frac{1}{6}t^6 - 1 \quad \text{for} \quad 0 \leq t \leq 2. \quad (\text{This one is a bit tricky. :})$$

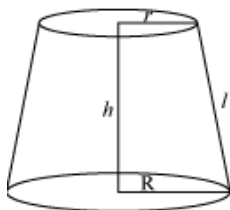
7.2 Areas of Surfaces of Revolution

Next, we want to find the surface area of a surface created by rotating a planar curve around the x -axis (so the surface created is hollow). We can find the approximate surface area and then use calculus to extend that to the exact surface area.

As is depicted in this picture, we can add up the surface area of lots of frustums with very small width (think of them like strips of paper or rubber bands wrapped around the surface).



But, what's the lateral surface area of a frustum?



I claim that the approximate surface area of a frustum (where the slant height is very small) is

$$SA \approx 2\pi \left(\frac{r+R}{2} \right) l$$

Why?

Then, the approximate surface area of the entire surface of revolution is given by:

And, the exact area is:

Ex 1: Find the surface area of the surface generated by revolving $y = \sqrt{16 - x^2}$ on the interval $[-1, 3]$.

Ex 2: Find the surface area of the surface generated by revolving $y = e^x$ on the interval $[0, 2]$.

Ex 3: Find the surface area of the surface generated by revolving $y=x^3$ on the interval $[1, 2]$.