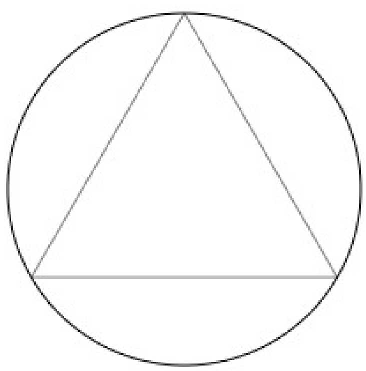
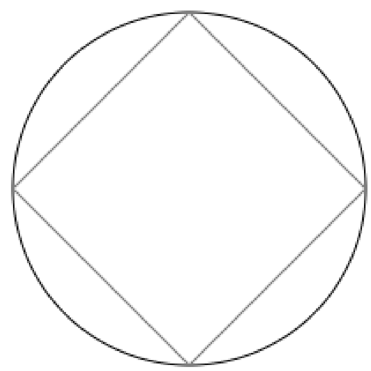
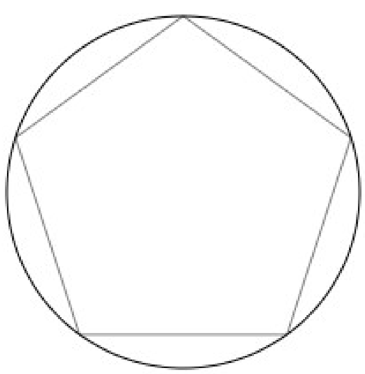
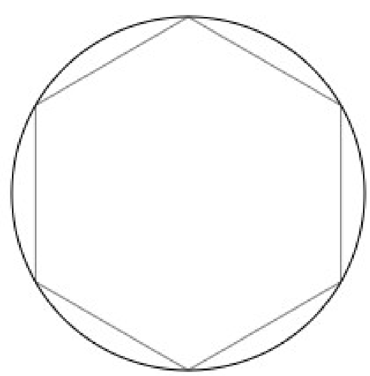
Math6100
Day 9 Notes
6.4 & 6.5, Area of Discs and Area of Regions bounded by Two Curves

<u>6.4 Area of Discs from Different Perspectives</u>

1. There are two circles, cut into sectors, on the last page of these notes. Cut each of the sectors out and tape the onto a page (as instructed in class).
   What is this activity trying to show?

2. Method of Exhaustion

Inscribe regular n-gons in a circle of radius r. Express the area of the n-gon A_n in terms of its apothem length a_n and perimeter P_n, i.e. assume a_n and P_n are given. Note: The apothem is the line segment connecting the center of the circle to the midpoint of the leg of the regular polygon. (There's a table provided on the next page.)

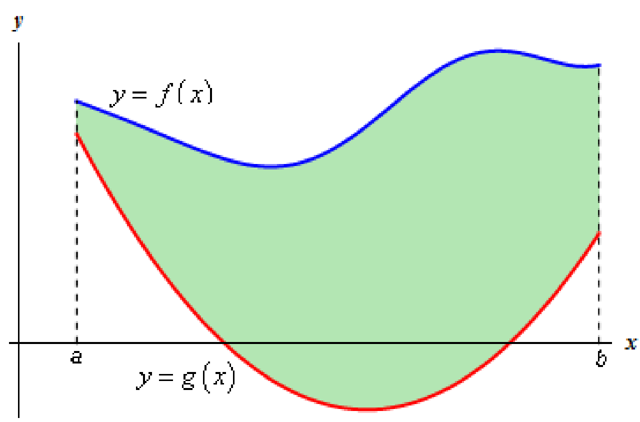
n	A_n
3	
4	
5	
6	
7	
8	
n	

$\lim_{n\to\infty} a_n =$ ____________________ $\lim_{n\to\infty} P_n =$ ____________________

This implies that $\lim_{n\to\infty} A_n =$ ____.

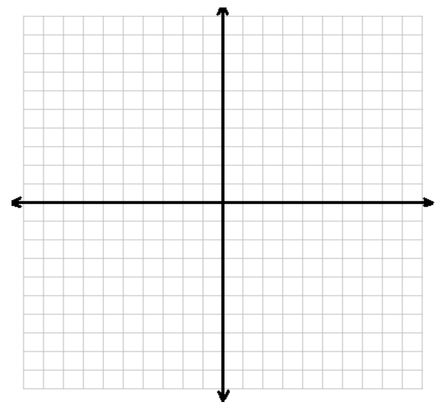
6.5 Area of Regions Bounded by Two Curves

How can we leverage what we know about definite integrals to help us find the area between two curves? (And, now we really want the actual area, not the signed area.)

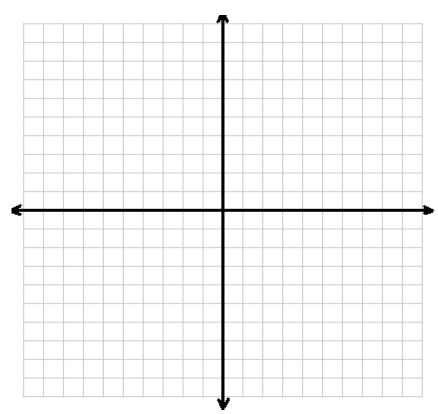


Ex 1: Find the area of these closed, bounded regions defined by the given curves.

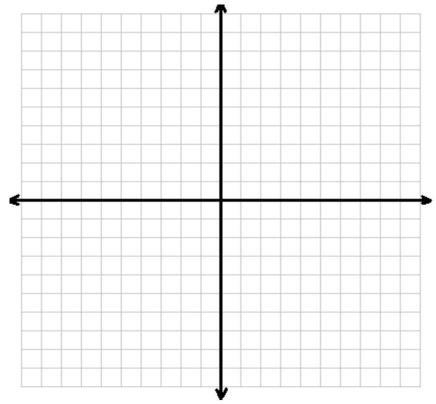
(a) $y=x^2$, $y=1-x$, $x=1$, $x=3$



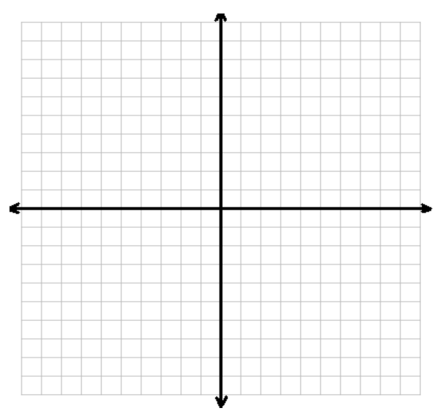
(b) $y=x^3-1$, $y=x-1$



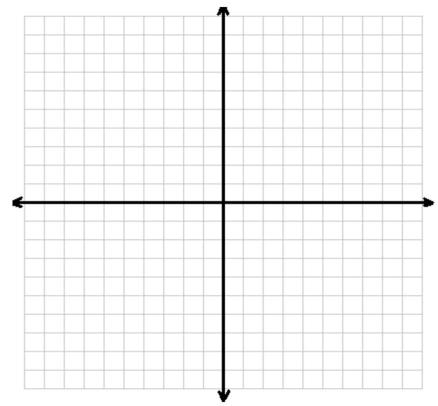
(c) $y=\sqrt{x-1}$, $y=0$ and $y=\sqrt{9-x}$



(d) $y=\frac{1}{3}x^2$ and $y=4-x^2$

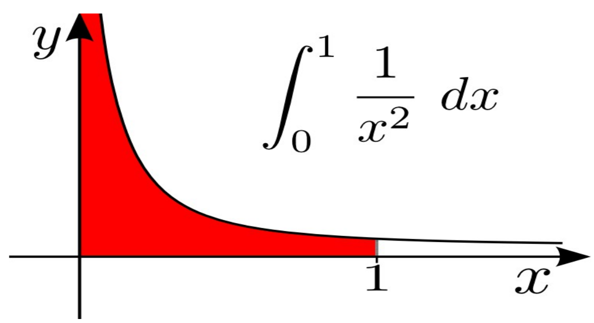


(e) $y=x^2$ and $y=2x-x^2$



Improper Integrals

What if we try to find the area across a vertical asymptote? Or find the area under the curve (between the curve and the x-axis) going off to infinity? Will the answer necessarily be infinite area?



Can we compute this integral?

Ex 2: Compute these improper integrals, or state that they diverge.

(a) $\int_0^1 \frac{1}{x^p}dx$ where p is a constant exponent. (You must consider all the cases.)

(b) $\int_{1}^{\infty} \frac{1}{x^{p}} dx$ where p is a constant exponent. (You must consider all the cases.)

(c) $\int_{-2}^{0} \frac{3x^5}{\sqrt[3]{(x^3+1)^4}}\, dx$

(d) $\int_{-\infty}^{0} 3(2x-1)^{-\frac{4}{3}}\, dx$